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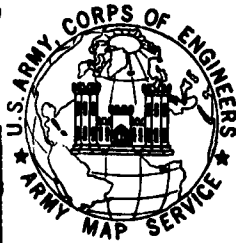


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ARMY MAP SERVICE

SEPTEMBER 1962

## TECHNICAL REPORT No. 45

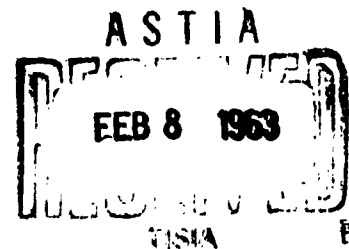
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## RANGE ERROR ANALYSIS

Alice G. Sturdivant

295842



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ARMY MAP SERVICE

TECHNICAL REPORT

NUMBER 45

RANGE ERROR ANALYSIS

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Mathematician, Army Map Service

Project No. MO-011  
September 1962



Submitted to  
THE CHIEF OF ENGINEERS, U. S. ARMY

By

The Commanding Officer

Army Map Service  
Washington 25, D. C.

## A B S T R A C T

This report presents the sample world-wide satellite tracking program carried out by the Army Map Service. By using satellites with known heights and inclinations and the coordinates of known points on geodetic datums, the coordinates of unknown points were related to these datums. The known and unknown points used in each segment of the program are presented, and the probable error in the correction to each unknown point is given.

This program is intended to be a theoretical problem for testing the range error analysis theory and equations and the resulting errors which would be associated with each determined position; therefore, the coordinates used for the known and unknown points were taken from maps and gazetteers and are only approximate.

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## RANGE ERROR ANALYSIS

### I. GENERAL PLAN

In order to determine as accurately as possible the coordinates of an unknown geodetic point on the earth in relation to other known points, satellites with known heights and inclinations are used. Since the absorption and refraction of the satellite signals increase with the observer's zenith distance, the accuracy of the signals received decreases accordingly; therefore, the satellite must necessarily be observed within a limited zenith distance. Applying this restriction three points on a known datum and a fourth point on an unknown datum are chosen, and the area of simultaneity is determined (Figure 1). As the distance between the points decreases, this area increases (Figure 2). However, the angle between the lines of sight from the ground stations to the satellite becomes so small that the accuracy of the information obtained is greatly reduced (Figure 3). For this reason the points must be chosen carefully.

Given the geometry set by the four stations, and with certain assumptions made regarding the ranging error, the error in the position of any one of the four stations can be determined. This is a simple application of the principle of propagation of errors that would occur in a planned world-wide triangulation net created by positioning at and tracking from four points (ranging stations) of the net at one time.

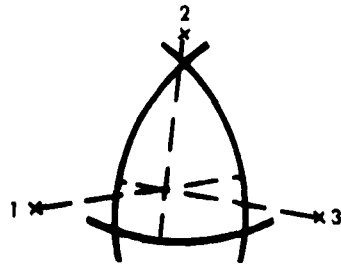


FIG. 1

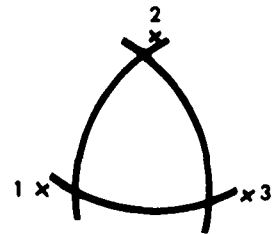
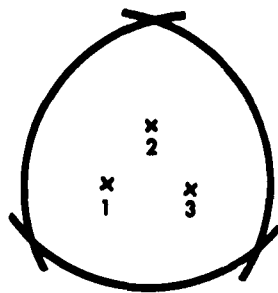


FIG. 2

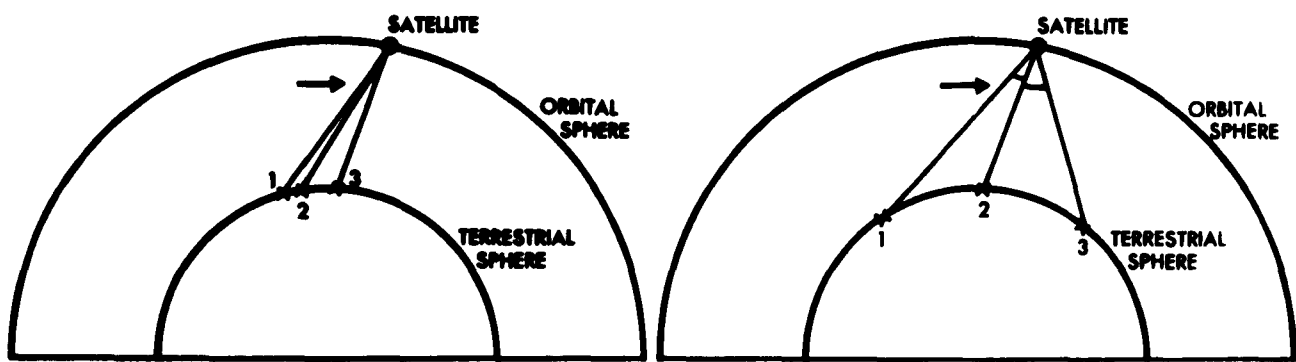


FIG. 3



## II. METHOD

In each configuration of four stations, the station coordinates and three satellite points chosen from within the area of simultaneity were incorporated into a series of matrix algebra solutions from which ultimately evolved the probable error in the corrections to the unknown station coordinates.

## III. EQUATIONS

The equations, their derivations, and the method of solution are as follows:

Satellite Positions ( $S^1_j, S^2_j, S^3_j$ ) and Station Positions ( $X^1_i, X^2_i, X^3_i$ ) are referred to an earth-fixed rectangular system with origin at the center of the reference ellipsoid,  $X^1(S^1)$  axis lying in the equator and directed positively toward the Greenwich Meridian,  $X^3(S^3)$  axis positive toward the North Pole and  $X^2(S^2)$  axis forming a right-handed system. The basis for the analysis is the distance relation

$$(r_{ji})^2 = (S^1_j - X^1_i)^2 + (S^2_j - X^2_i)^2 + (S^3_j - X^3_i)^2$$

which yields the error equation

$$\Delta r_{ji} = \sum_{k=1}^3 \frac{S^k_j - X^k_i}{r_{ji}} (\Delta S^k_j - \Delta X^k_i)$$

with J satellite positions observed from each of four stations (station #4 assumed "unknown"). These equations may be arranged as follows:

$$\begin{aligned} \left[ \Delta r_{ji} \right] &= Q_1 \left[ \Delta s_j^k \right] - Q_2 \left[ \Delta x_1^k \right] \\ k &= 1, 2, 3 \\ i &= 1, 2, 3 \\ j &= 1, 2, \dots, J \end{aligned}$$

$$\begin{aligned} \left[ \Delta r_{j4} \right] &= Q_3 \left[ \Delta s_j^k \right] - Q_4 \left[ \Delta x_4^k \right] \\ k &= 1, 2, 3 \\ j &= 1, 2, \dots, J \end{aligned}$$

where:

$$\begin{bmatrix} \Delta x_4^1 \\ \Delta x_4^2 \\ \Delta x_4^3 \end{bmatrix}, \begin{bmatrix} \Delta r_{j1} \\ \Delta r_{j2} \\ \Delta r_{j3} \\ \Delta r_{j4} \end{bmatrix} = \begin{bmatrix} \Delta r_{11} \\ \Delta r_{12} \\ \Delta r_{13} \\ \Delta r_{21} \\ \vdots \\ \Delta r_{J3} \end{bmatrix}, \begin{bmatrix} \Delta s_j^1 \\ \Delta s_j^2 \\ \vdots \\ \Delta s_j^3 \end{bmatrix} = \begin{bmatrix} \Delta x_1^1 \\ \Delta x_1^2 \\ \vdots \\ \Delta x_3^2 \\ \Delta x_3^3 \end{bmatrix}$$

and the  $Q_n$  are shown on the following pages.

Choosing three suitably located satellite positions and assuming the variances in ranges and "known" station coordinates are known, the final relation  $M \sigma \Delta^2 M^T$  gives the variances in the corrections to the "unknown" station coordinates.

#### Variance Matrix of Point Located By Ranging

Given:  $s_j^k$ ,  $k = 1, 2, 3$  and  $j = 1, 2, \dots, J$

and  $x_1^k$ ,  $K = 1, 2, 3$  and  $i = 1, 2, 3, 4$ ,

compute for each pair (j i):

$$r_{ji} = \sqrt{\sum_k (s_j^k - x_i^k)^2} \text{ and for each set } (j \ i \ k):$$

$$q_{jik} = \frac{s_j^k - x_i^k}{r_{ji}}$$

Form the matrices  $Q_1, Q_2, Q_3, Q_4$ :

$$Q_1 = \begin{bmatrix} q_{111} & q_{112} & q_{113} & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ q_{121} & q_{122} & q_{123} & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ q_{131} & q_{132} & q_{133} & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & q_{211} & q_{212} & q_{213} & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & q_{221} & q_{222} & q_{223} & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & q_{231} & q_{232} & q_{233} & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & q_{J11} & q_{J12} & q_{J13} \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & q_{J21} & q_{J22} & q_{J23} \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & q_{J31} & q_{J32} & q_{J33} \end{bmatrix}$$

$$Q_2 = \begin{bmatrix} q_{111} & q_{112} & q_{113} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & q_{121} & q_{122} & q_{123} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & q_{131} & q_{132} & q_{133} \\ q_{211} & q_{212} & q_{213} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & q_{221} & q_{222} & q_{223} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & q_{231} & q_{232} & q_{233} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ q_{J11} & q_{J12} & q_{J13} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & q_{J21} & q_{J22} & q_{J23} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & q_{J31} & q_{J32} & q_{J33} \end{bmatrix}$$

$$Q_3 = \begin{bmatrix} q_{141} & q_{142} & q_{143} & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & q_{241} & q_{242} & q_{243} & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & q_{341} & q_{342} & q_{343} & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & q_{J41} & q_{J42} & q_{J43} \end{bmatrix}$$

$$Q_4 = \begin{bmatrix} q_{141} & q_{142} & q_{143} \\ q_{241} & q_{242} & q_{243} \\ q_{341} & q_{342} & q_{343} \\ q_{441} & q_{442} & q_{443} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ q_{J41} & q_{J42} & q_{J43} \end{bmatrix}$$

The dimensions of matrices  $Q_1, Q_2, Q_3, Q_4$  are as follows:

<u>Matrix</u>	<u>Rows</u>	<u>Columns</u>
$Q_1$	$3J$	$3J$
$Q_2$	$3J$	$9$
$Q_3$	$J$	$3J$
$Q_4$	$J$	$3$

Compute the matrices:

$$\begin{aligned}
 C &= \begin{bmatrix} Q_4^T & Q_4 \end{bmatrix}^{-1} Q_4^T = \{c_{mn}\} \\
 B &= C Q_3 \begin{bmatrix} Q_1^T & Q_1 \end{bmatrix}^{-1} Q_1^T = \{b_{mn}\} \\
 A &= B Q_2 = \{a_{mn}\}
 \end{aligned}$$

and form the matrix

$$M = \begin{bmatrix} A & B & C \end{bmatrix}$$

Compute:

$$\{\sigma^2 x_4\} = M \sigma \Delta^2 M^T$$

where the elements of  $\sigma \Delta^2$  are given.

The dimensions of matrices  $A, B, C, M, \{\sigma^2 x_4\}, \sigma \Delta$  are as follows:

<u>Matrix</u>	<u>Rows</u>	<u>Columns</u>
$A$	$3$	$9$
$B$	$3$	$3J$
$C$	$3$	$J$
$M$	$3$	$4J + 9$
$\{\sigma^2 x_4\}$	$3$	$3$
$\sigma \Delta$	$4J + 9$	$4J + 9$

The derivation of the equations is as follows:

$$\begin{bmatrix} \Delta r_{ji} \end{bmatrix} = Q_1 \begin{bmatrix} \Delta s_j^k \end{bmatrix} - Q_2 \begin{bmatrix} \Delta x_i^k \end{bmatrix} \quad \text{I}$$

$$\begin{bmatrix} Q_1^T Q_1 \end{bmatrix}^{-1} Q_1^T \begin{bmatrix} \Delta r_{ji} \end{bmatrix} + \begin{bmatrix} Q_1^T Q_1 \end{bmatrix}^{-1} Q_1^T Q_2 \begin{bmatrix} \Delta x_i^k \end{bmatrix} = \begin{bmatrix} \Delta s_j^k \end{bmatrix} \quad \text{II}$$

$$\begin{bmatrix} \Delta r_{jl} \end{bmatrix} = Q_3 \begin{bmatrix} \Delta s_j^k \end{bmatrix} - Q_4 \begin{bmatrix} \Delta x_l^k \end{bmatrix} \quad \text{III}$$

$$- \begin{bmatrix} Q_4^T Q_4 \end{bmatrix}^{-1} Q_4^T \begin{bmatrix} \Delta r_{jl} \end{bmatrix} + \begin{bmatrix} Q_4^T Q_4 \end{bmatrix}^{-1} Q_4^T Q_3 \begin{bmatrix} \Delta s_j^k \end{bmatrix} = \begin{bmatrix} \Delta x_l^k \end{bmatrix} \quad \text{IV}$$

Substituting II in IV and setting

$$C = \begin{bmatrix} Q_4^T Q_4 \end{bmatrix}^{-1} Q_4^T$$

$$B = C Q_3 \begin{bmatrix} Q_1^T Q_1 \end{bmatrix}^{-1} Q_1^T$$

$$A = B Q_2$$

the equation becomes:

$$\begin{bmatrix} \Delta x_l^k \end{bmatrix} = A \begin{bmatrix} \Delta x_i^k \end{bmatrix} + B \begin{bmatrix} \Delta r_{ji} \end{bmatrix} - C \begin{bmatrix} \Delta r_{jl} \end{bmatrix}$$

$$\begin{bmatrix} \Delta x_l^k \end{bmatrix} = \begin{bmatrix} A & B & -C \end{bmatrix} \begin{bmatrix} \Delta x_i^k \\ \Delta r_{ji} \\ \Delta r_{jl} \end{bmatrix}$$

Setting

$$\begin{bmatrix} A & B & -C \end{bmatrix} = M, \quad \begin{bmatrix} \Delta x_i^k \\ \Delta r_{ji} \\ \Delta r_{jl} \end{bmatrix} = \Delta$$

Then  $\sigma \Delta^2 x_l = M \sigma \Delta^2 M^T$  = the variances in the corrections to the "unknown" station coordinates.

#### IV. RESULTS TABLES

The variances (probable errors) computed for 73 "unknown" station coordinates ranged from  $\pm 89.4$  meters to  $\pm 589.6$  meters. Of the 73 variances computed, 55 (or 75%) were in the interval of  $\pm 100$  to  $\pm 300$  meters. There were only 2 variances less than  $\pm 100$  meters. The following table shows the distribution of the variances and the final results.

<u>INTERVAL</u> <u>(Meters)</u>	0 - 99	100-199	200-299	300-399	400-499	500-599
NO. OF VARIANCES	2	32	23	7	5	4

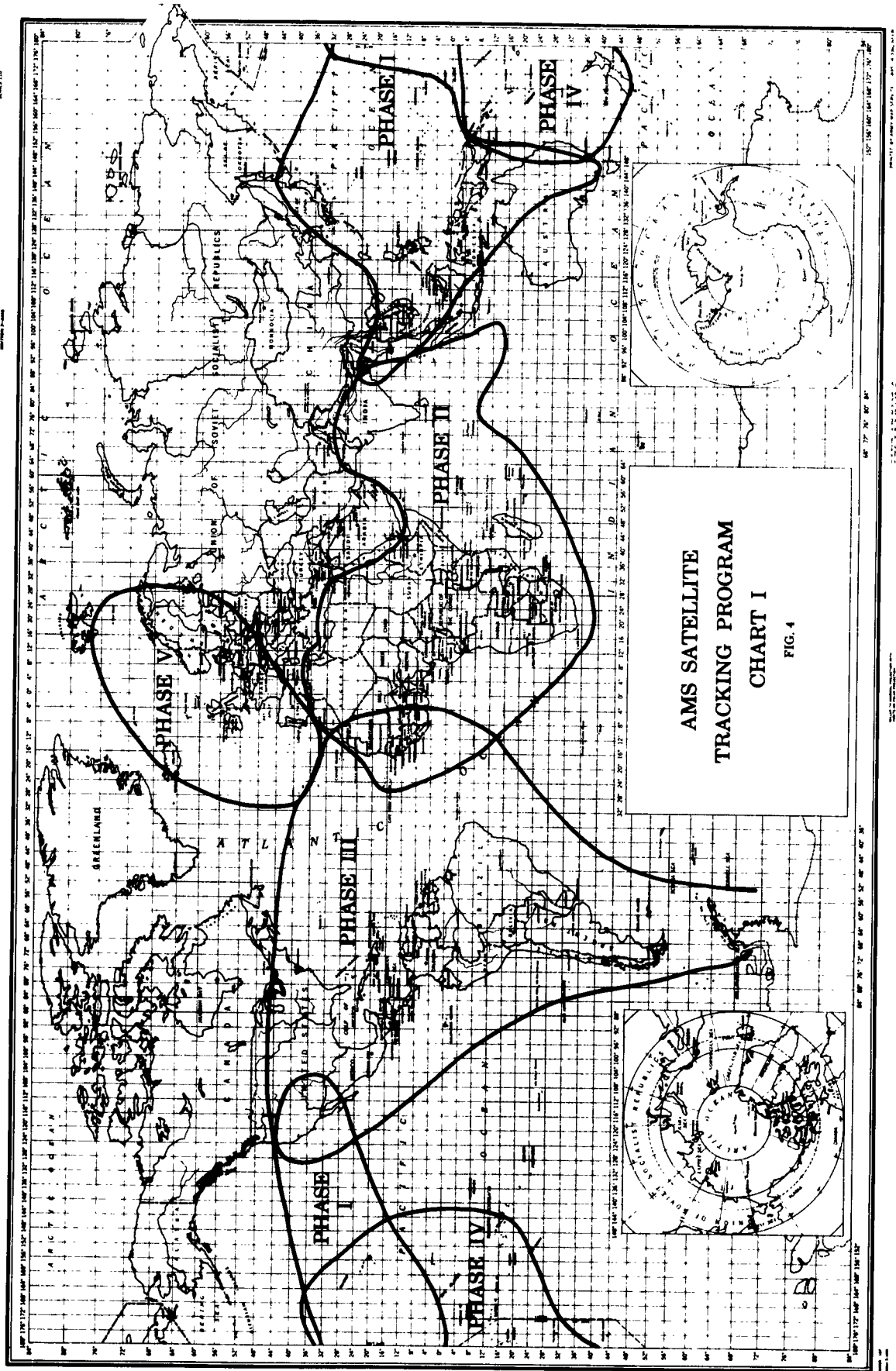
The station coordinates, satellite positions, and plotted areas of simultaneity used for each configuration in the error analysis are shown on the following pages. See Figure 4 for the world-wide program.

THE WORLD 1:60,000,000



THE WORLD

THE WORLD 1:60,000,000  
SCALE 1:60,000,000





SATELLITE A

50° Inclination

Height = 1104 KM

Phase I - 1

Johnston, Kauai, Laysan - - - Tarawa

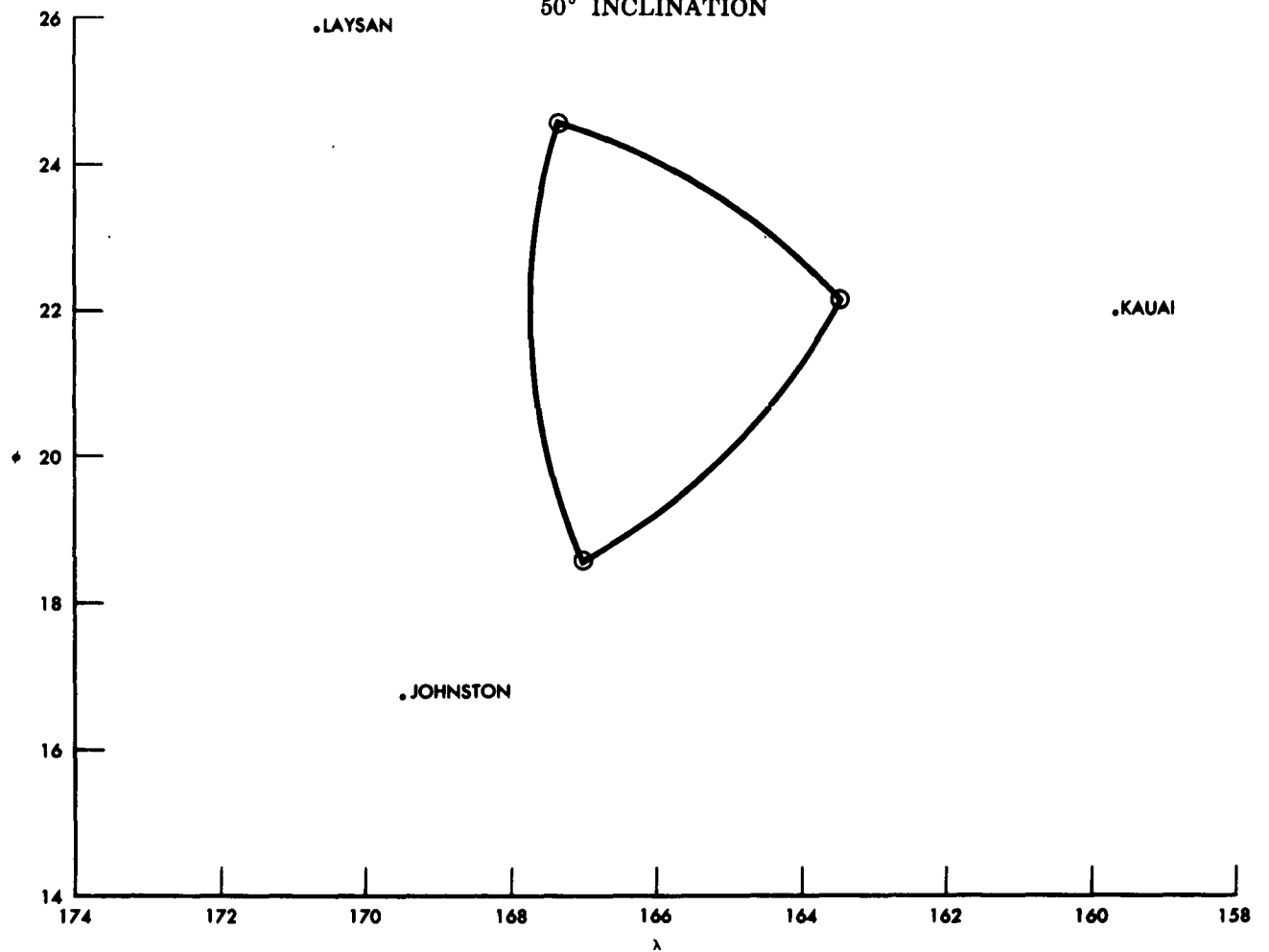
	$\phi$	$\lambda$	h(meters)	Name	
1	16.75	-169.51667	12	Johnston	Known Stations
2	21.96667	-159.6667	6	Kauai	
3	25.76667	-171.73333	11	Laysan	
4	1.38333	173.15	3	Tarawa	Unknown
I	7.8	178.0	1104000		Satellite Positions Fixed by Unknown Stations
II	-6.0	176.0	1104000		
III	2.4	165.2	1104000		
I	24.4	-167.3	1104000		Satellite Positions Fixed by Known Stations
II	22.1	-163.6	1104000		
III	18.7	-167.0	1104000		

Limit 60° Zenith Distance

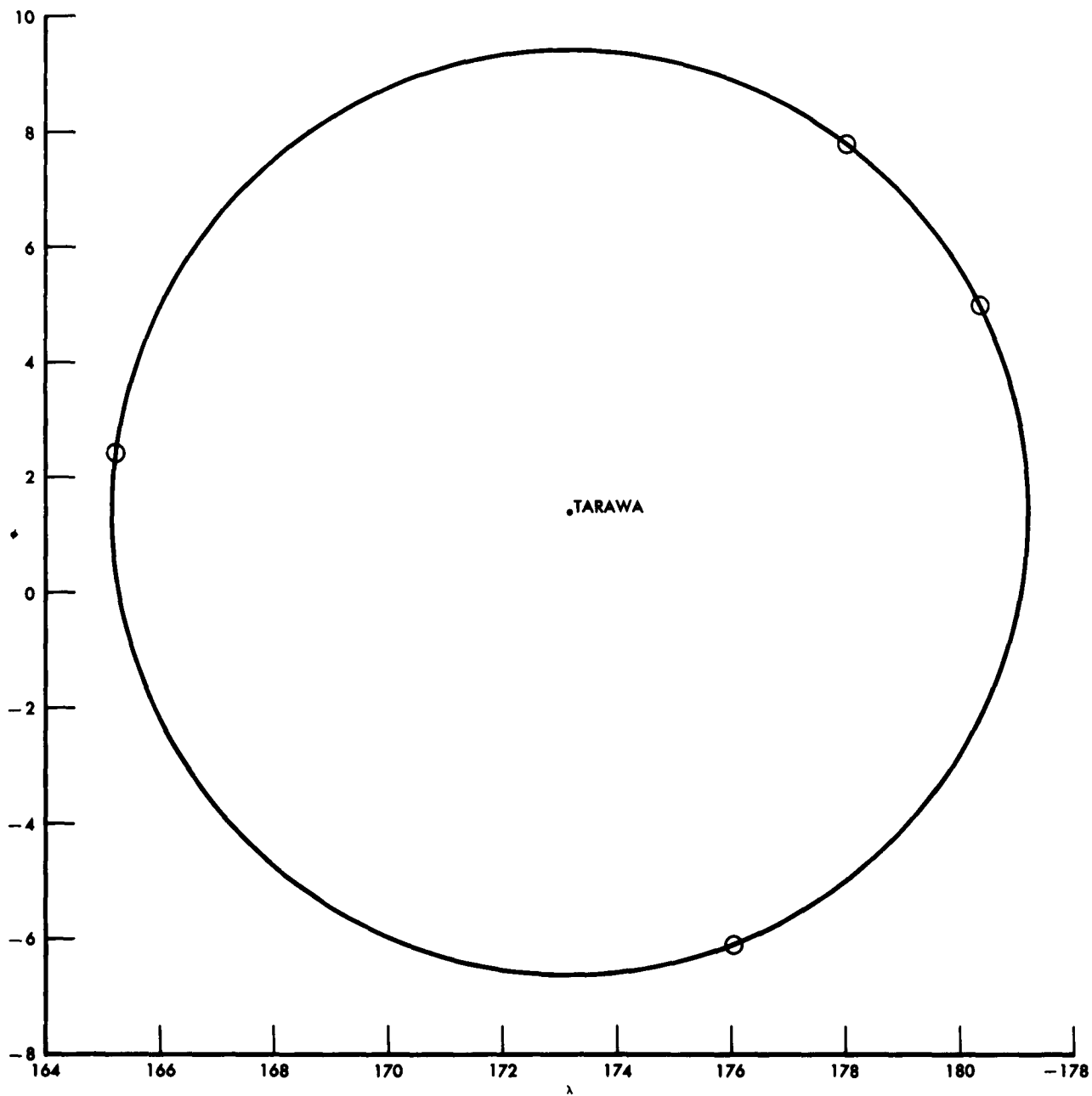
FINAL RESULTS:

Tarawa  $\pm$  99.3 Uncertainty (Meters)

SATELLITE A, PHASE I-1  
 JOHNSTON, KAUAI, LAYSAN-TARAWA  
 (KNOWN) (UNKNOWN)  
 SATELLITE POSITIONS FIXED BY KNOWN STATIONS  
 LIMIT 60° ZENITH DISTANCE  
 50° INCLINATION



SATELLITE A, PHASE I-1  
 JOHNSTON, KAUAI, LAYSAN-TARAWA  
 (KNOWN) (UNKNOWN)  
 SATELLITE POSITIONS FIXED BY UNKNOWN STATION  
 LIMIT 60° ZENITH DISTANCE  
 50° INCLINATION



SATELLITE A 50° Inclination  
Height = 1104 KM

Phase I - 2

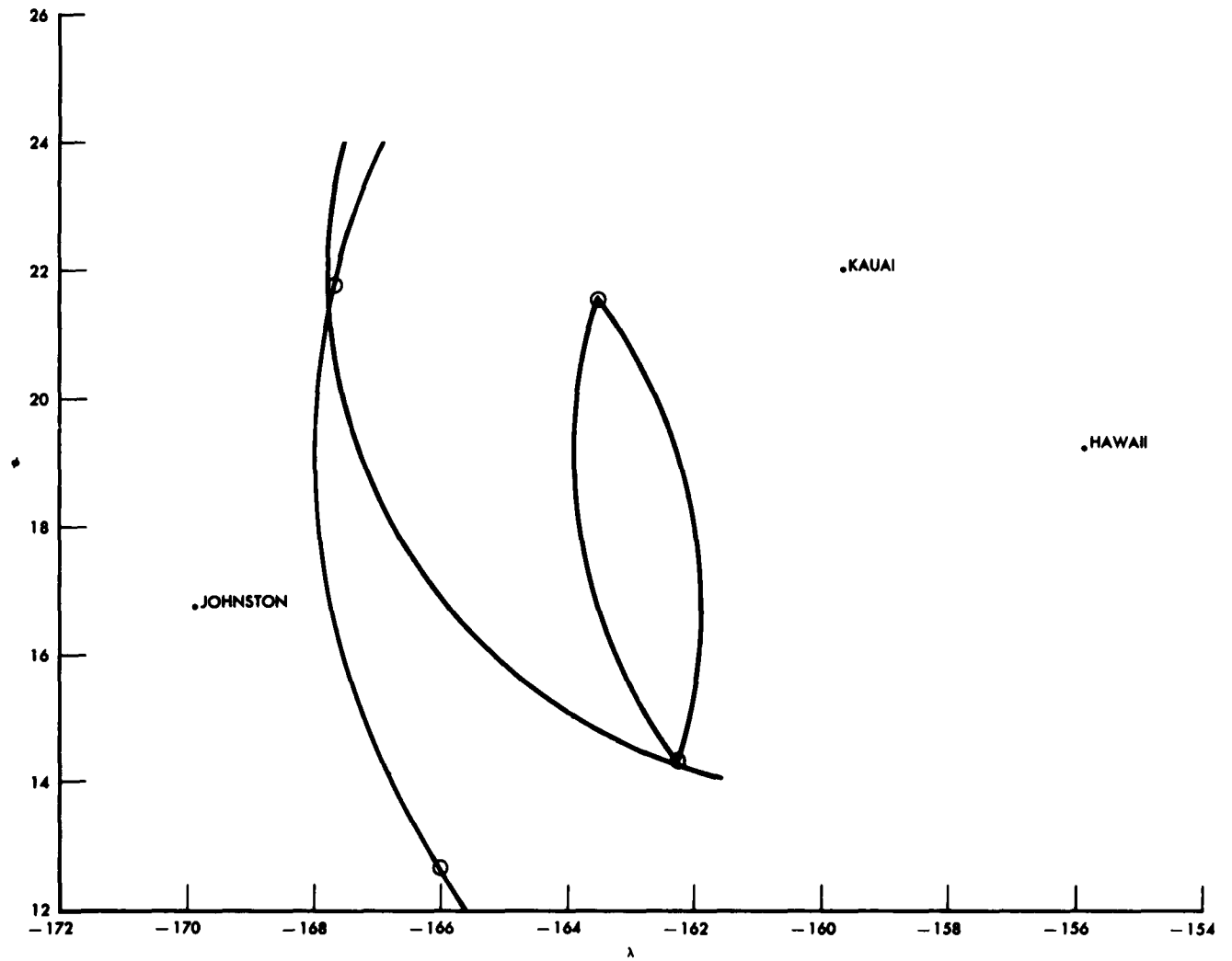
Johnston, Kauai, Hawaii --- Palmyra, Howland

	$\phi$	$\lambda$	h(meters)	Name	
1	16.75	-169.51667	12	Johnston	Known Stations
2	21.96667	-159.66667	6	Kauai	
3	19.21667	-155.86667	457	Hawaii	
4	5.86667	-162.1	2	Palmyra	Unknown Station
I	16.5	-167.5	1104000		Satellite Positions Limit 60° Zenith Distance
II	16.9	-157.5	1104000		
III	10.2	-162.0	1104000		
4	0.8	-176.63333	9	Howland	Unknown Station
I	4.2	176.2	1104000		Satellite Positions Fixed by Unknown Station
II	-6.4	180.0	1104000		
III	3.8	-169.2	1104000		
I	21.8	-167.6	1104000		Satellite Positions Fixed by Known Stations
II	21.5	-163.5	1104000		
III	14.4	-162.2	1104000		

FINAL RESULTS:

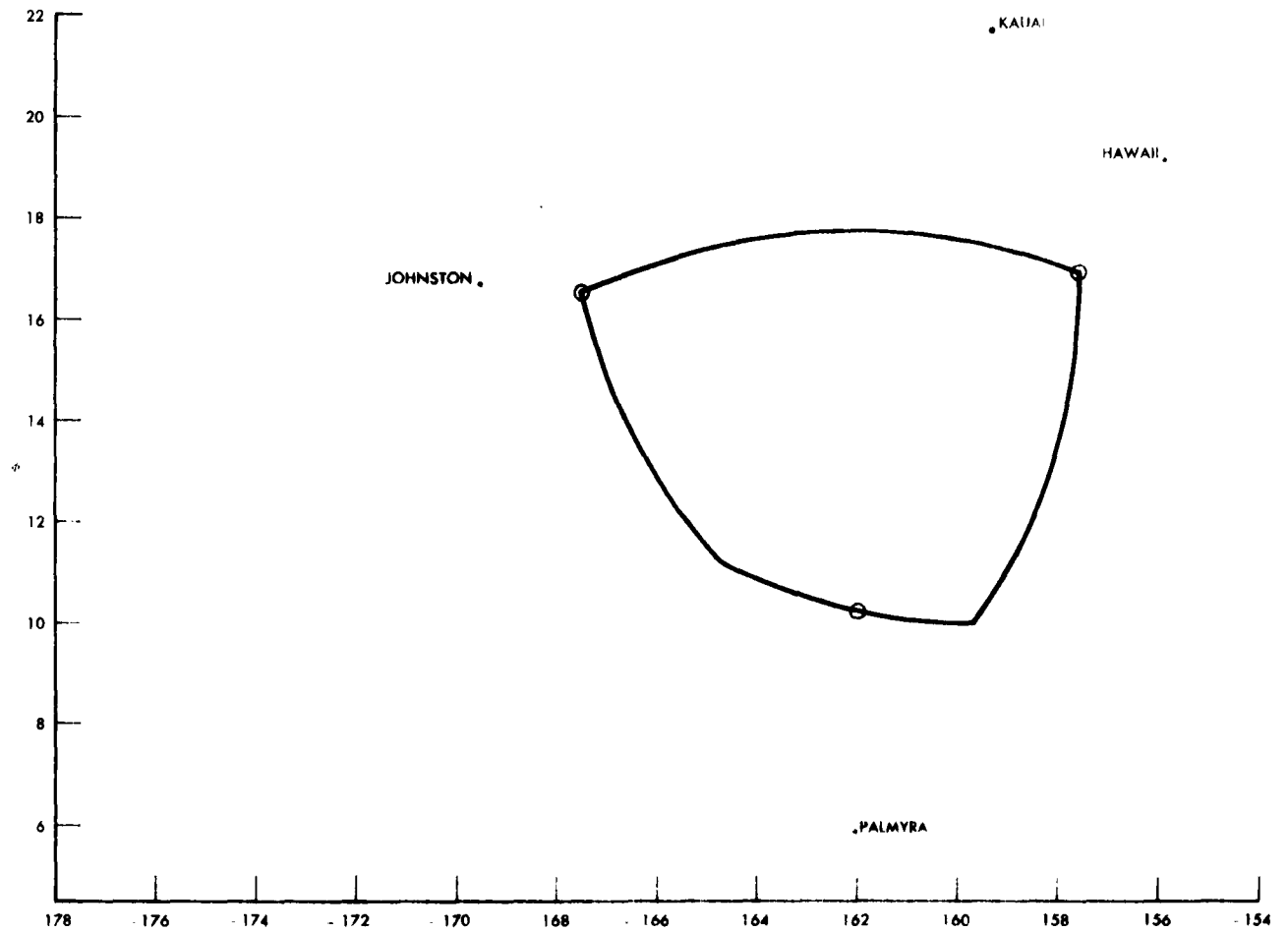
Palmyra  $\pm 584.8$  Uncertainty (meters)  
Howland  $\pm 157.8$  " "

SATELLITE A, PHASE I-2  
 LIMIT 60° ZENITH DISTANCE  
 SATELLITE POSITIONS FIXED BY KNOWN STATIONS  
 (JOHNSTON, KAUAI, HAWAII) FOR ORBITAL  
 OBSERVATION WITH UNKNOWN (HOWLAND)



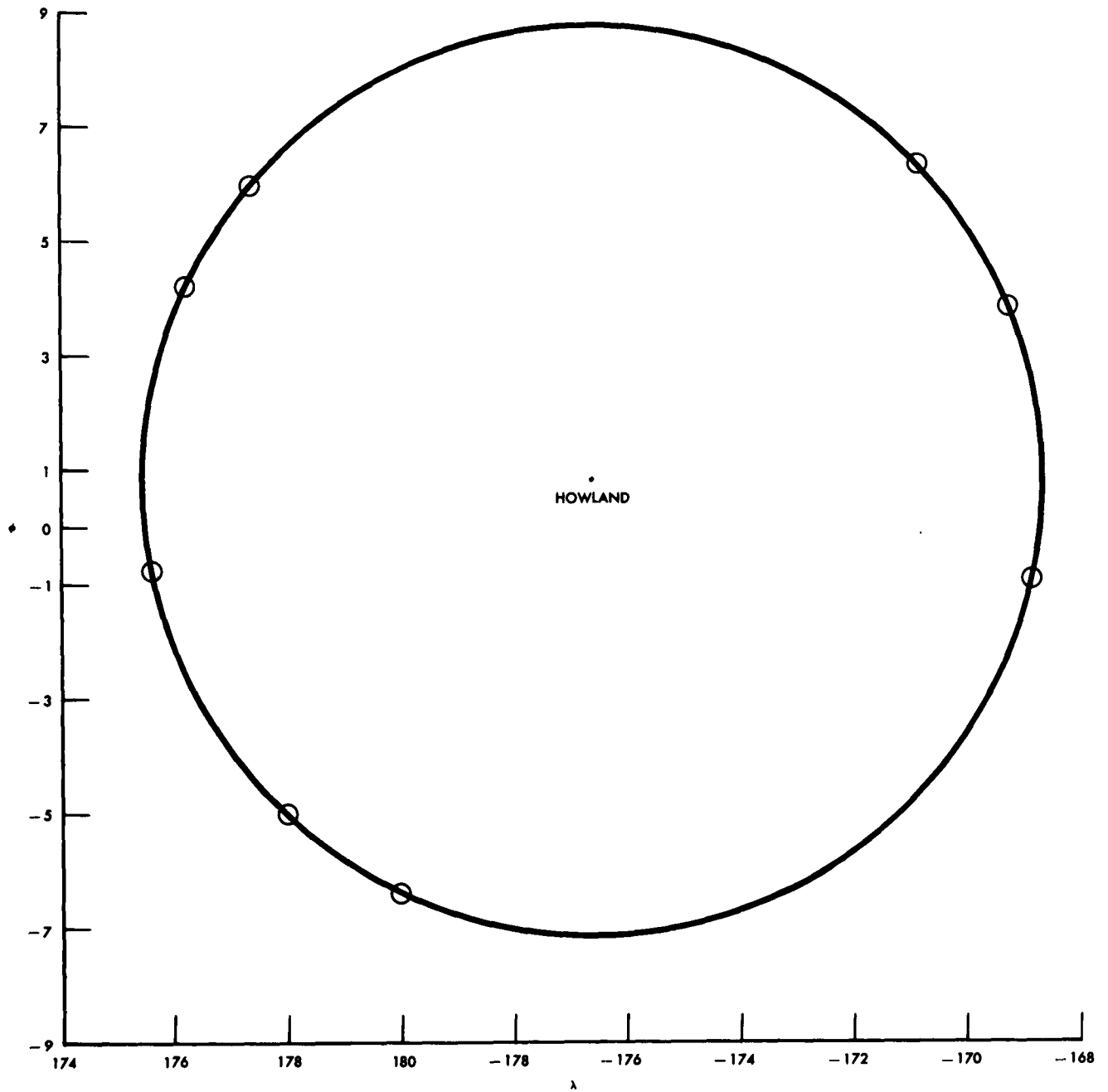
SATELLITE A, PHASE 12  
JOHNSTON, KAUAI, HAWAII-  
(KNOWN)  
PALMYRA  
(UNKNOWN)

SATELLITE POSITIONS FIXED BY KNOWN STATIONS  
WITH UNKNOWN STATION FOR SIMULTANEOUS OBSERVATION  
LIMIT 60° ZENITH DISTANCE



SATELLITE A, PHASE 1-2  
JOHNSTON, KAUAI, HAWAII--  
(KNOWN)  
HOWLAND  
(UNKNOWN)

SATELLITE POSITIONS FIXED BY UNKNOWN  
STATION FOR ORBITAL OBSERVATION  
LIMIT 60° ZENITH DISTANCE



SATELLITE A      50° Inclination  
Height = 1104 KM

Phase I - 3

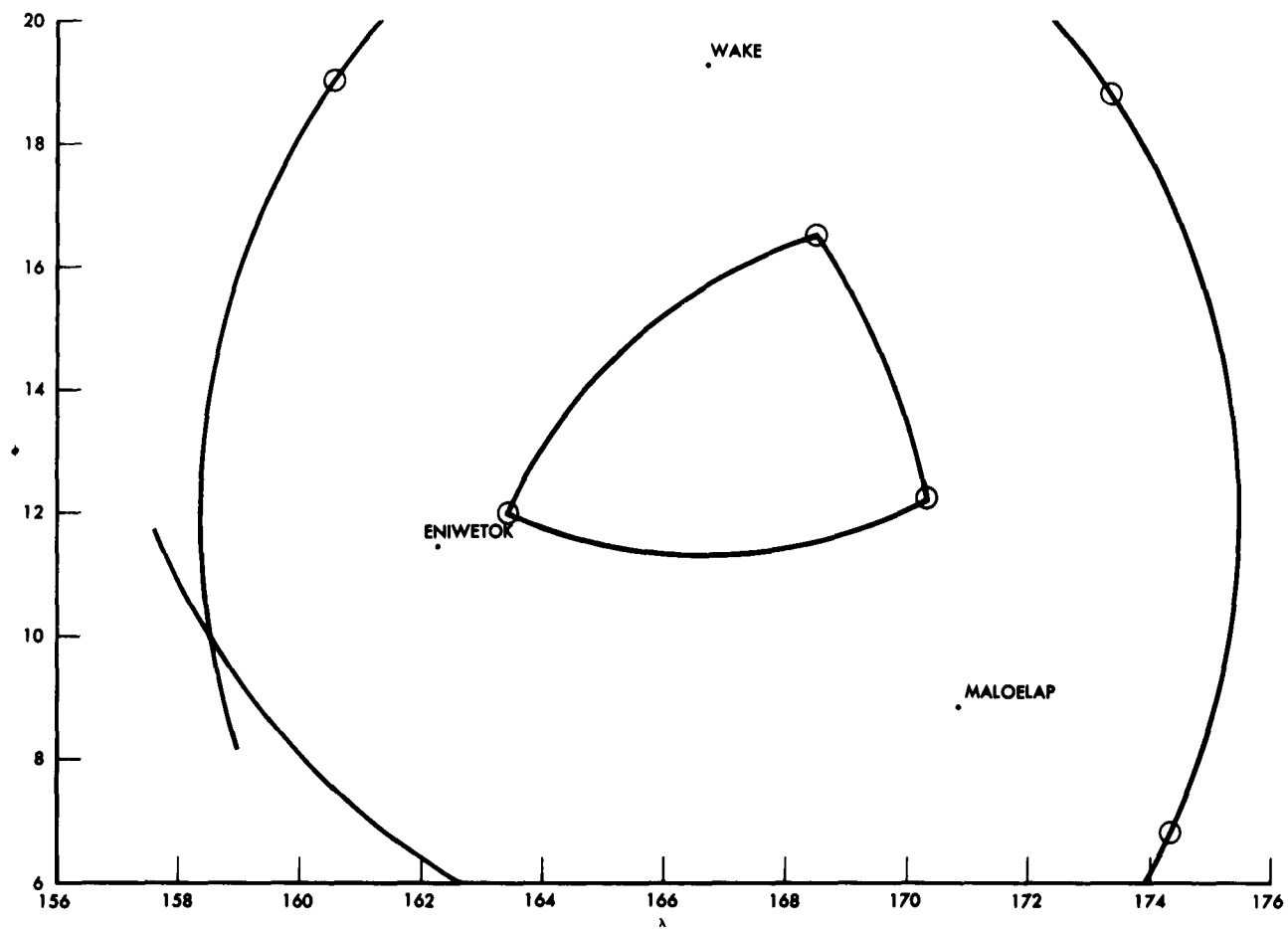
Wake, Eniwetok, Maloelap --- Midway, Howland					
	$\phi$	$\lambda$	h(meters)	Name	
1	19.26667	166.65	6.0	Wake	Known Stations
2	11.35	162.33333	5.0	Eniwetok	
3	8.9	170.85	4.0	Maloelap	
4	0.8	-176.63333	9.0	Howland	Unknown Station
I	-5.0	178.0	1104000		Satellite Positions Fixed by Unknown Station
II	6.2	176.4	1104000		
III	-1.0	-168.8	1104000		
I	12.0	163.5	1104000		Satellite Positions Fixed by Known Stations
II	16.5	168.5	1104000		
III	12.1	170.3	1104000		
4	28.21667	-177.33333	3	Midway	Unknown Station
I	22.6	176.9	1104000		Satellite Positions Fixed by Unknown Station
II	32.7	176.0	1104000		
III	29.0	-167.2	1104000		

FINAL RESULTS:

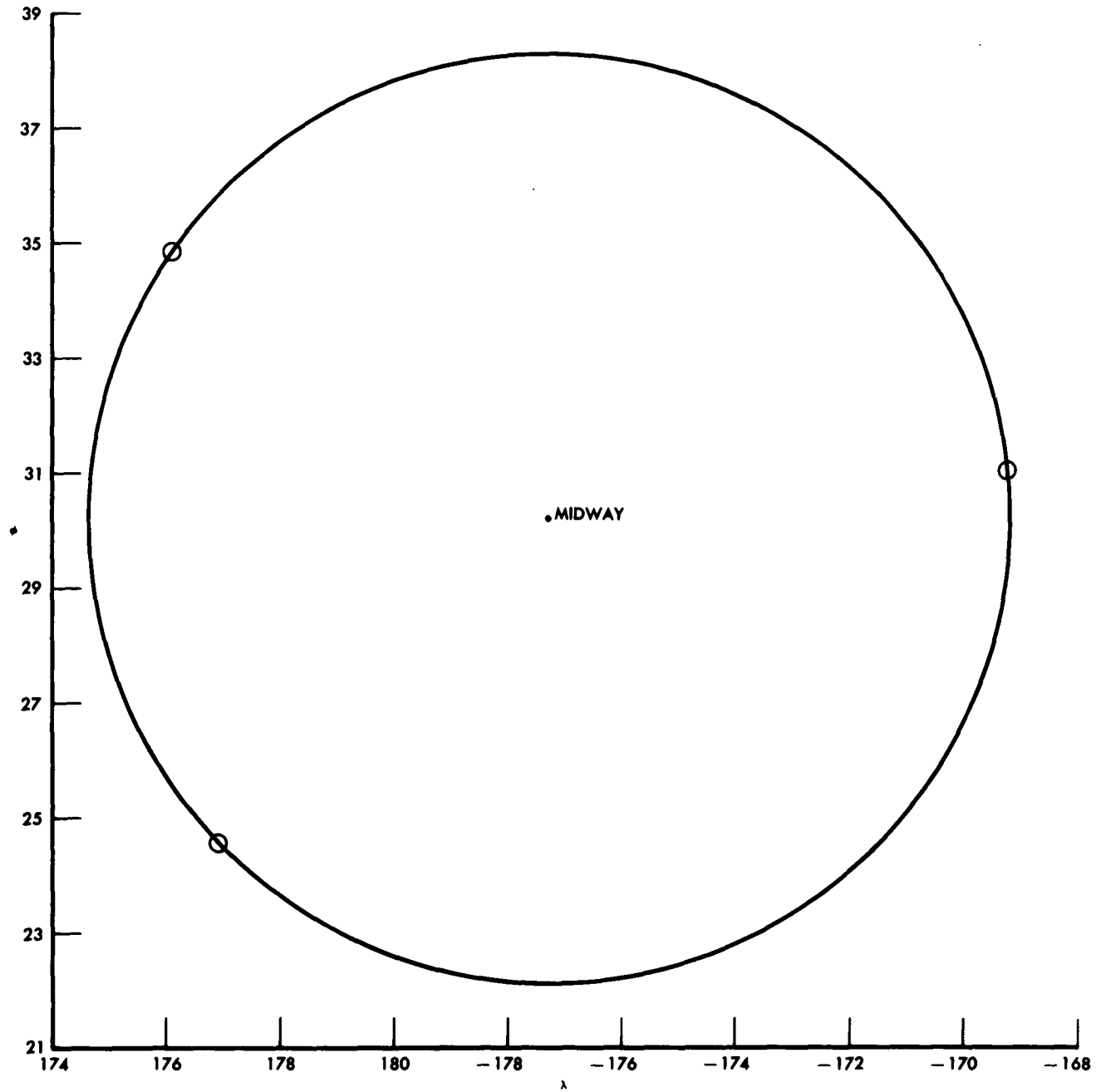
Howland     $\pm$  195.4    Uncertainty (meters)  
Midway      $\pm$  102.2    "                "



SATELLITE A, PHASE I-3  
 WAKE, ENIWETOK, MALOELAP  
 (KNOWN)  
 LIMIT 60° ZENITH DISTANCE  
 SATELLITE POSITIONS FIXED BY KNOWN  
 STATIONS FOR ORBITAL OBSERVATION



SATELLITE A, PHASE I-3  
 WAKE, ENIWETOK, MALOELAP—  
 (KNOWN)  
 MIDWAY  
 (UNKNOWN)  
 LIMIT 60° ZENITH DISTANCE  
 SATELLITE POSITION FIXED BY UNKNOWN  
 STATION FOR ORBITAL OBSERVATION



50° Inclination  
 SATELLITE A  
 Height = 1104 KM

Phase I - 4

Wake, Eniwetok, Maloelap --- Tarawa, Nauru, Kusaie

	$\phi$	$\lambda$	h(meters)	Name	
1	19.26667	166.65	6.0	Wake	
2	11.35	162.33333	5.0	Eniwetok	Known Stations
3	8.9	170.85	4.0	Maloelap	
4	1.38333	173.15	3.0	Tarawa	Unknown Station
I	13.2	174.0	1104000		
II	7.9	163.0	1104000		Satellite Positions
III	9.8	174.0	1104000		
4	-0.53333	166.91667	8.0	Nauru	Unknown Station
I	9.4	173.5	1104000		
II	9.3	160.0	1104000		Satellite Positions
III	7.3	166.9	1104000		
4	5.31667	163.0	629.0	Kusaie	Unknown Station
I	9.8	174.0	1104000		
II	17.2	162.2	1104000		Satellite Positions
III	10.1	158.9	1104000		

**FINAL RESULTS:**

Tarawa	$\pm$ 407.1	Uncertainty (meters)	
Nauru	$\pm$ 521.8	"	"
Kusaie	$\pm$ 160.0	"	"

50° Inclination  
SATELLITE A  
Height = 1104 KM

Phase I - 5

Wake, Eniwetok, Maloelap - - - Ponape, Tokyo, Marcus					
	$\phi$	$\lambda$	h(meters)	Name	
1	19.26667	166.65	6.0	Wake	
2	11.35	162.33333	5.0	Eniwetok	Known Stations
3	8.9	170.85	4.0	Maloelap	
4	6.96667	158.2	305.0	Ponape	Unknown Station
I	10.1	158.9	1104000		
II	7.8	170.1	1104000		Satellite Positions
III	18.0	163.1	1104000		
I	12	163.5	1104000		Satellite Positions
II	16.5	168.5	1104000		Fixed by Known Stations
III	12.1	170.3	1104000		
4	35.65	139.75	100	Tokyo	Unknown Station
I	30.6	146.0	1104000		Satellite Positions
II	40.6	146.0	1104000		Fixed by Unknown Station
III	37.4	132.0	1104000		
4	24.3	153.96667	7	Marcus	Unknown Station
I	20.2	160.8	1104000		Satellite Positions
II	29.8	148.4	1104000		Fixed by Unknown Station
III	21.2	146.6	1104000		

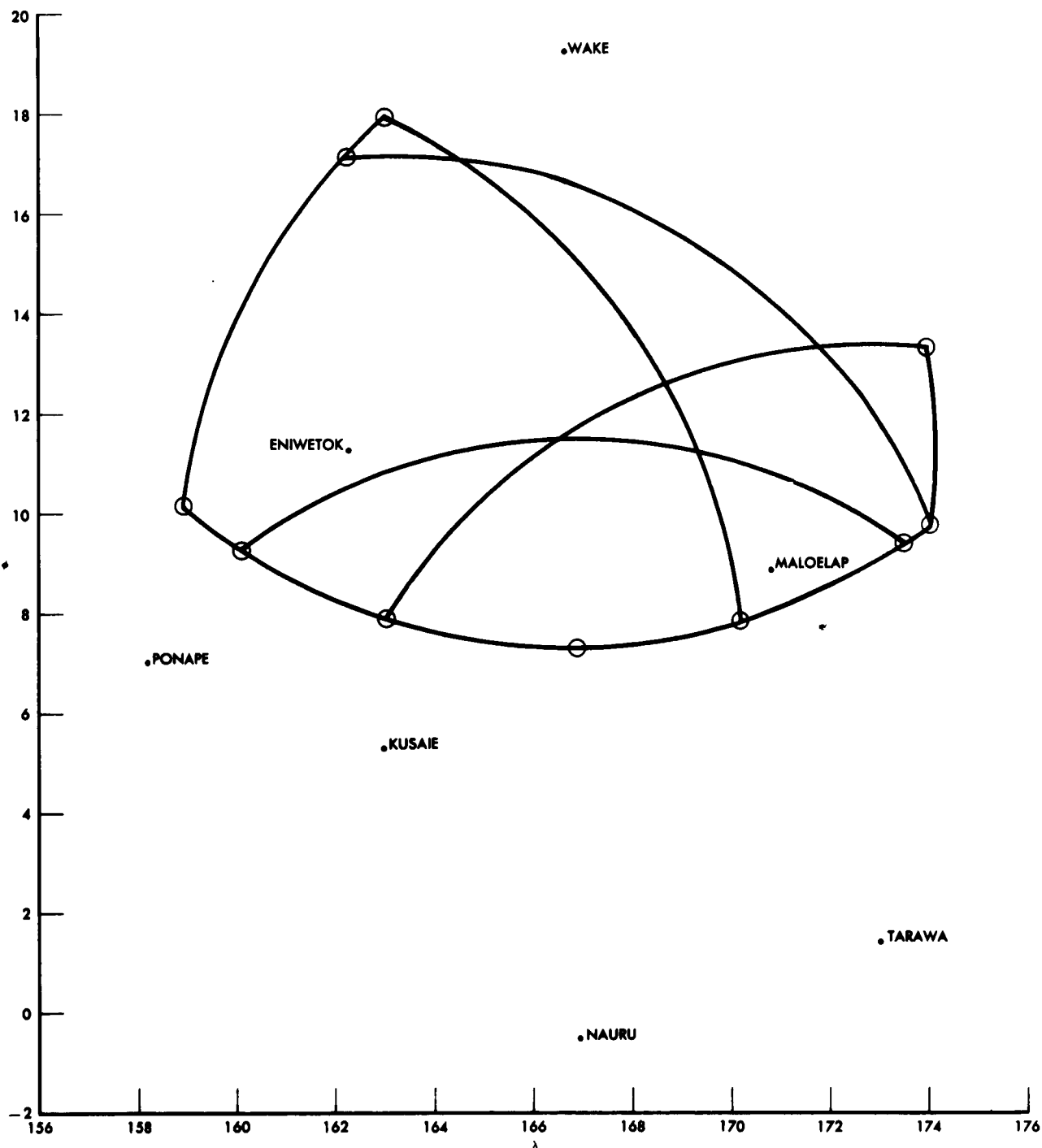
FINAL RESULTS:

Ponape	$\pm$ 159.0	Uncertainty (meters)	
Tokyo	$\pm$ 103.4	"	"
Marcus	$\pm$ 141.8	"	"

SATELLITE A, PHASE I-4, 5  
 LIMIT 60° ZENITH DISTANCE  
 WAKE, ENIWETOK, MALOELAP--  
 (KNOWN)

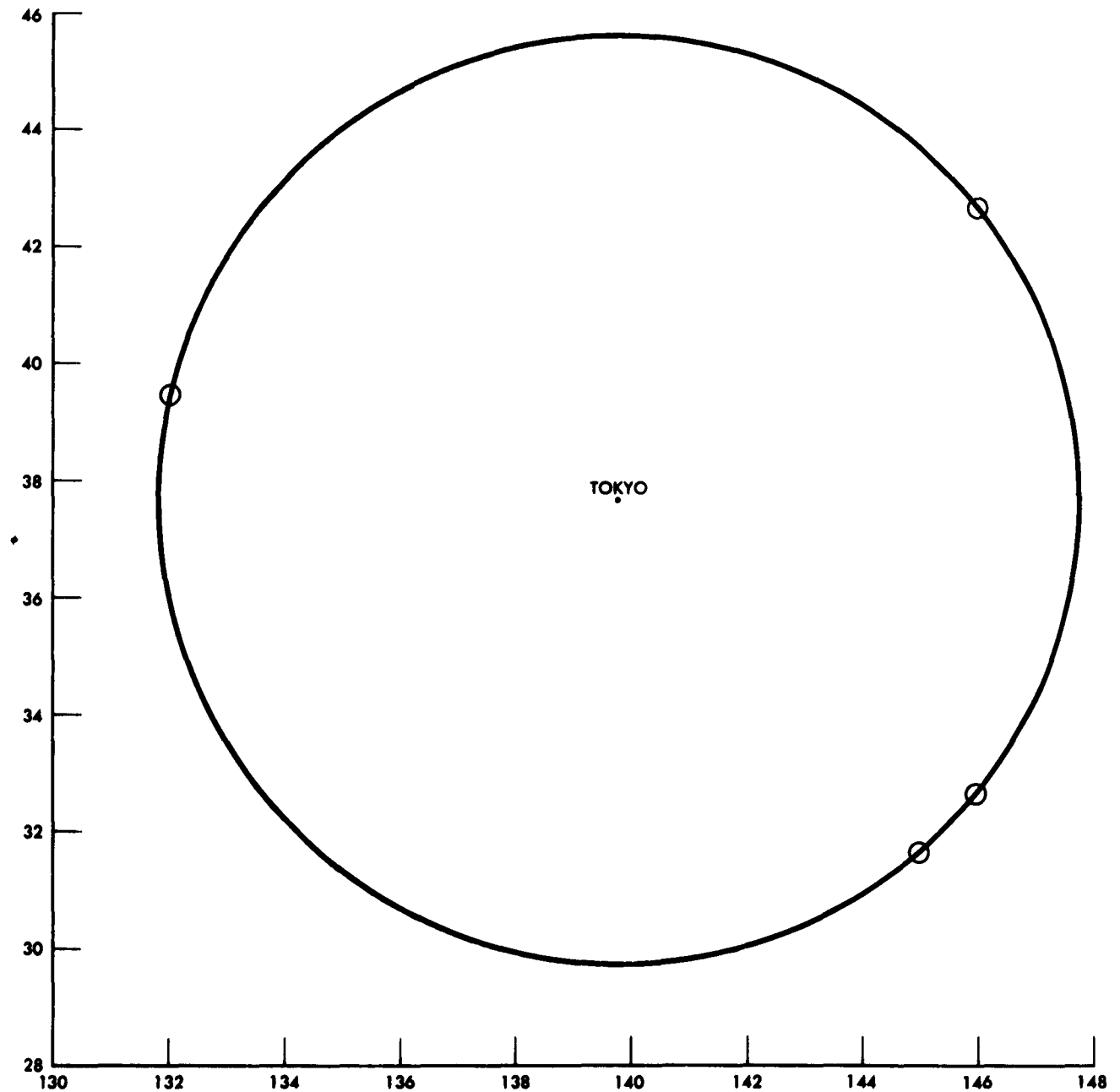
TARAWA, NAURU, KUSAIE, PONAPE  
 (UNKNOWN)

STATION COORDINATES AND SATELLITE POSITIONS  
 FIXED BY KNOWN AND UNKNOWN STATIONS  
 FOR SIMULTANEOUS OBSERVATIONS



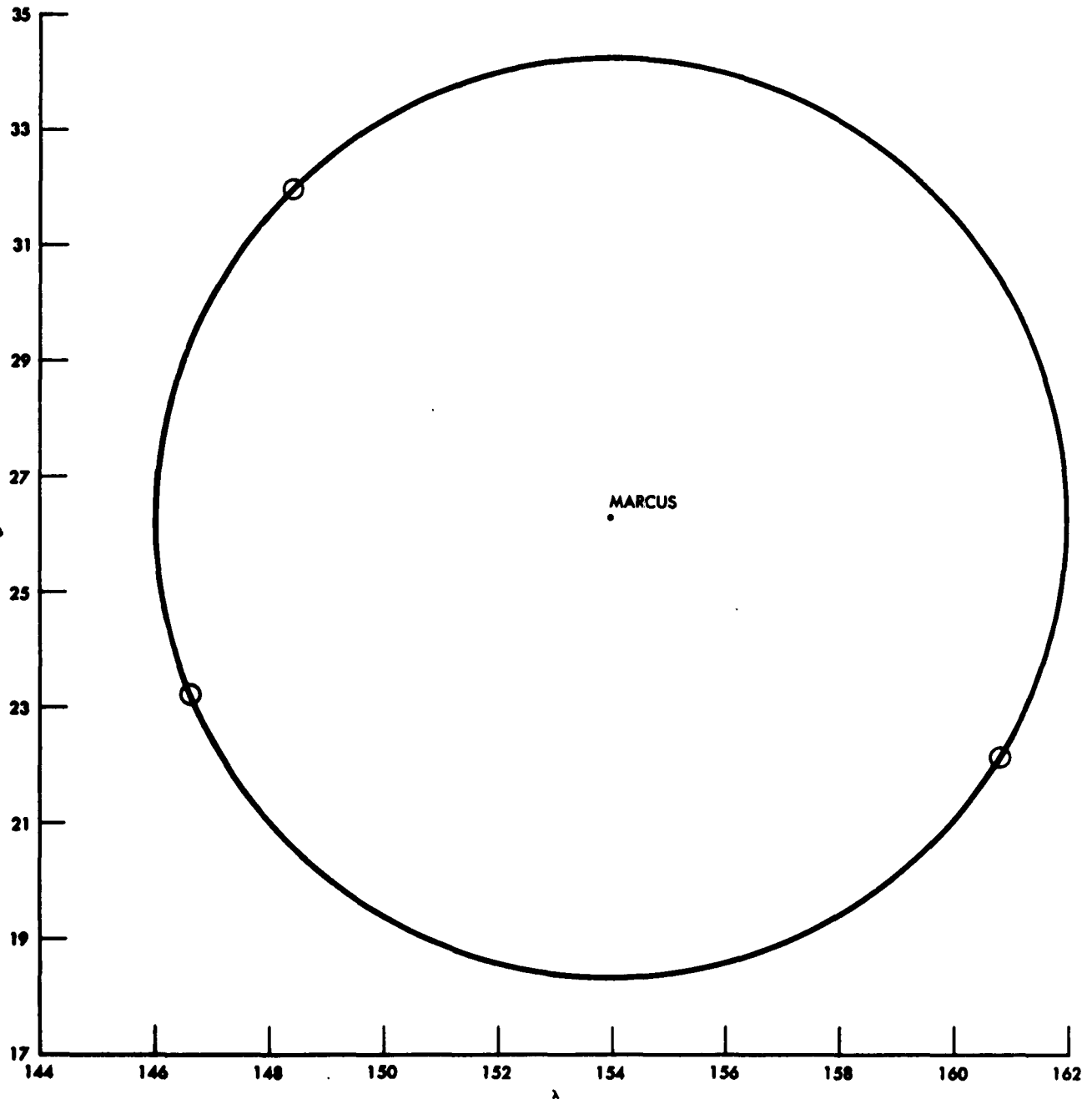
SATELLITE A, PHASE I-5  
LIMIT 60° ZENITH DISTANCE  
WAKE, ENIWETOK, MALOELAP-  
(KNOWN)  
TOKYO  
(UNKNOWN)

SATELLITE POSITIONS FIXED BY UNKNOWN  
STATION FOR ORBITAL OBSERVATION



SATELLITE A, PHASE I-5  
LIMIT 60° ZENITH DISTANCE  
WAKE, ENIWETOK, MALOELAP—  
(KNOWN)  
MARCUS  
(UNKNOWN)

SATELLITE POSITION FIXED BY UNKNOWN  
STATION FOR ORBITAL OBSERVATION



SATELLITE A      50° Inclination  
 Height = 1104 KM

Phase I - 6

Kusaie, Ponape, Eniwetok --- Truk

	$\phi$	$\lambda$	h(meters)	<u>Name</u>	
1	5.31667	163.0	629	Kusaie	
2	6.96667	158.2	305	Ponape	Known Stations
3	11.35	162.33333	5	Eniwetok	
<hr/>					
4	7.45	151.85	370	Truk	Unknown Station
<hr/>					
I	6.9	151.2	1104000		
II	16.7	159.6	1104000		Satellite Positions
III	-0.6	160.8	1104000		
<hr/>					

FINAL RESULTS:

Truk  $\pm$  229.2    Uncertainty (meters)



SATELLITE C      40° Inclination  
 Height = 1104 KM

Phase I - 7

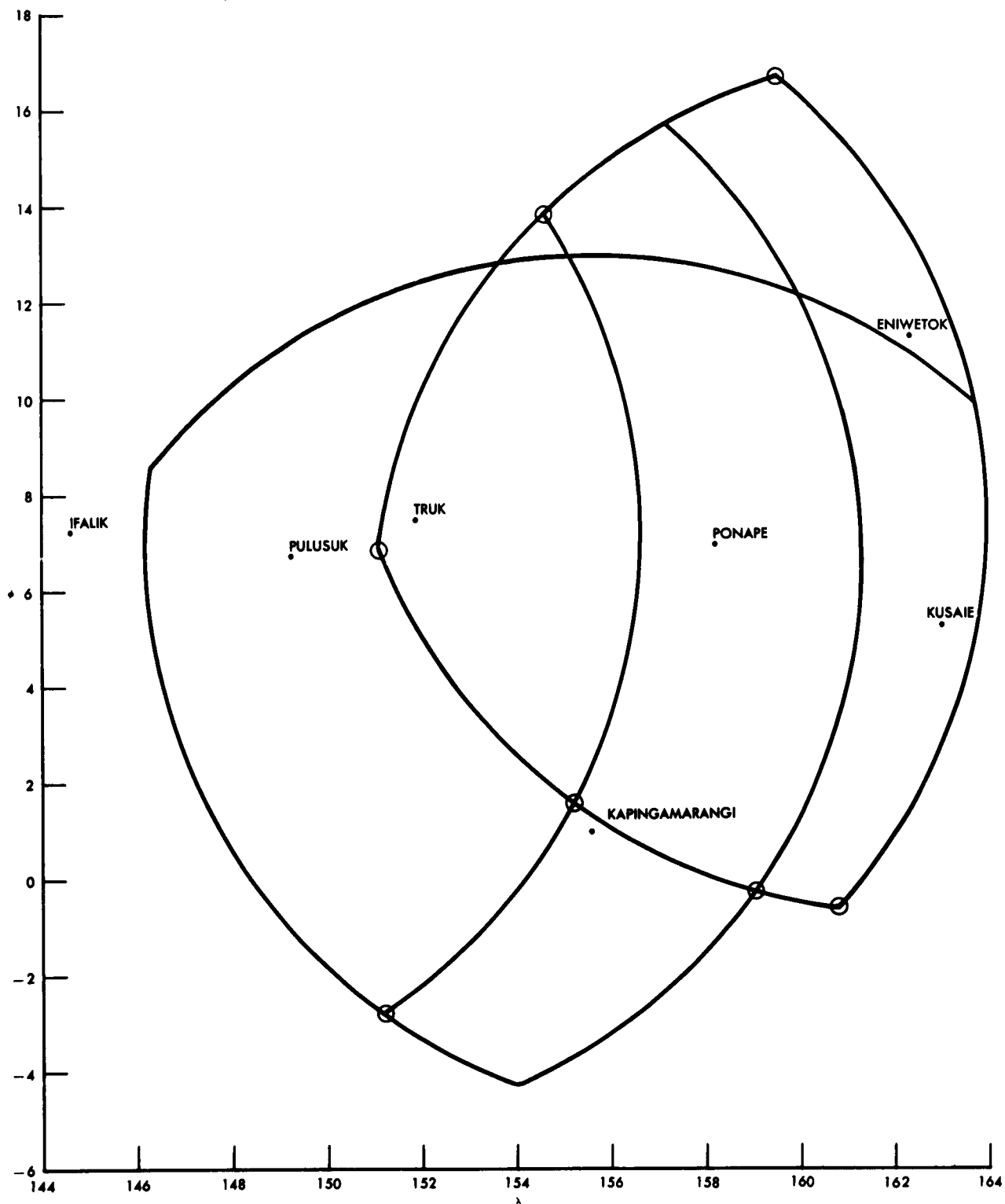
Truk, Ponape, Kapingamarangi --- Pulusuk, Ifalik

	$\phi$	$\lambda$	h(meters)	<u>Name</u>	
1	7.45	151.85	370	Truk	
2	6.96667	158.2	305	Ponape	Known Stations
3	1.06667	155.75	25	Kapingamarangi	
4	6.68333	149.31667	50	Pulusuk	Unknown Station
I	8.6	146.4	1104000		
II	-4.3	154.3	1104000		Satellite Positions
III	12.2	159.9	1104000		
4	7.2333	144.36667	27	Ifalik	Unknown Station
I	8.6	146.4	1104000		
II	-2.7	151.2	1104000		Satellite Positions
III	13.0	155.1	1104000		

FINAL RESULTS:

Pulusuk    ±   126.2    Uncertainty (meters)  
 Ifalik     ±   262.1    "            "

SATELLITE A, PHASE I-6,7  
 LIMIT 30° ZENITH DISTANCE  
 KUSAIE, PONAPE, ENIWETOK—PHASE I-6 (TRUK)  
 (KNOWN) (UNKNOWN)  
 TRUK, PONAPE, KAPINGAMARANGI—PHASE I-7 (PULUSUK, IFALIK)  
 (KNOWN) (UNKNOWN)  
 SATELLITE POSITIONS FIXED BY KNOWN AND  
 UNKNOWN STATIONS FOR SIMULTANEOUS OBSERVATIONS



SATELLITE C

40° Inclination

Height = 1104 KM

Phase I - 8

Truk, Ifalik, Manus			- - - -	Saipan, Guam, Ulithi	
	$\phi$	$\lambda$		h(meters)	Name
1	7.45	151.85		370	Truk
2	7.23333	144.36667		27	Ifalik Known Stations
3	-1.91667	146.88333		1	Manus
4	15.2	145.7		152	Saipan Unknown Station
I	4.7	140.3		1104000	
II	7.2	154.4		1104000	Satellite Positions
III	9.9	147.0		1104000	
4	13.48333	144.78333		100	Guam Unknown Station
I	2.2	141.2		1104000	
II	7.0	154.8		1104000	Satellite Positions
III	9.9	147.6		1104000	
4	10.03333	139.78333		1	Ulithi Unknown Station
I	-1.1	143.7		1104000	
II	7.7	140.0		1104000	Satellite Positions
III	9.0	151.6		1104000	

FINAL RESULTS:

Saipan  $\pm$  281.9 Uncertainty (meters)

Guam  $\pm$  215.4 " "

Ulithi  $\pm$  193.9 " "

SATELLITE C      40° Inclination  
                          Height = 1104 KM

Phase I - 9

Truk, Ifalik, Manus - - - Palau, New Guinea

	$\phi$	$\lambda$	h(meters)	Name	
4	7.61667	134.58333	183	Babelthuap, Palau	Unknown Station
I	-0.6	143.2	1104000		
II	7.7	140.0	1104000		Satellite Positions
III	9.9	146.2	1104000		
4	-1.48333	137.9	30	Kaap D'Urville, New Guinea	Unknown Station
I	-3.4	149.6	1104000		
II	0.6	142.2	1104000		Satellite Positions
III	9.2	142.9	1104000		

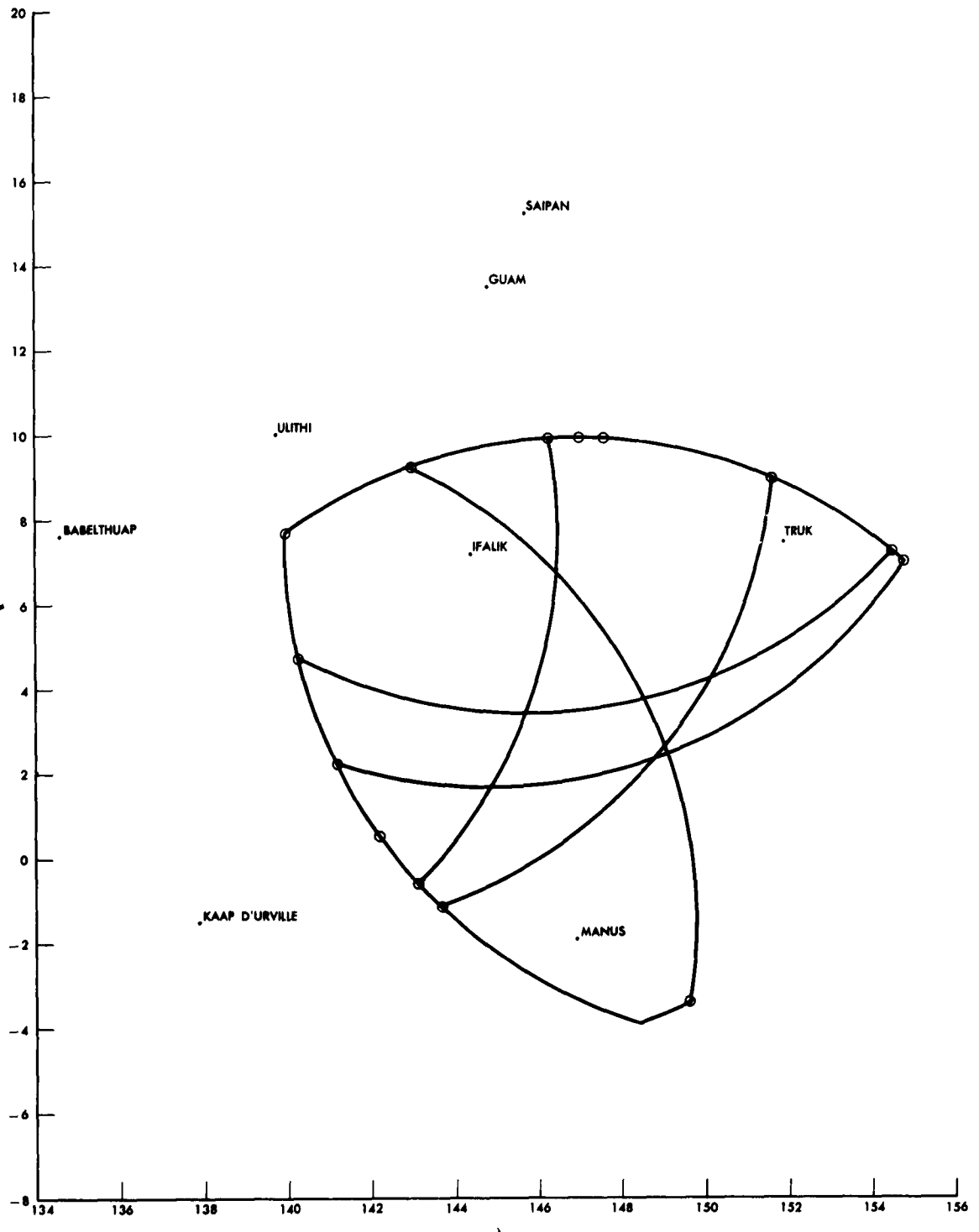
FINAL RESULTS:

Palau             $\pm$  369.9    Uncertainty (meters)  
 New Guinea     $\pm$  297.2       "               "

SATELLITE C, PHASE I-8, 9  
 LIMIT 60° ZENITH DISTANCE  
 TRUK, IFALIK, MANUS—  
 (KNOWN)

PHASE I-8 (SAIPAN, GUAM, ULITHI)  
 PHASE I-9 (PALAU, NEW GUINEA)  
 (UNKNOWN)

SATELLITE POSITIONS FIXED BY KNOWN  
 AND UNKNOWN STATIONS FOR SIMULTANEOUS OBSERVATIONS.



40° Inclination  
Height = 1104 KM

Phase I - 10

Kapingamarangi, Rabaul, Manus -- Port Moresby, Cooktown,  
Townsville

	$\phi$	$\lambda$	h(meters)	Name	
1	1.66667	155.75	25	Kapingamarangi	
2	-4.2	152.18333	16	Rabaul	Known Stations
3	-1.91667	146.88333	2	Manus	
4	-9.46667	147.2	152	Port Moresby	Unknown Station
I	1.8	143.9	1104000		Satellite Positions
II	-6.3	158.4	1104000		Fixed by Unknown Station
III	-7.2	148.2	1104000		
4	-15.46667	145.33333	100	Cooktown	Unknown Station
I	-3.8	147.6	1104000		Satellite Positions
II	-23.7	154.0	1104000		Fixed by Unknown Station
III	-26.5	138.0	1104000		
4	-19.5	146.66667	100	Townsville	Unknown Station
I	-9.8	155.9	1104000		Satellite Positions
II	-0.4	144.0	1104000		Fixed by Unknown Station
III	1.0	160.6	1104000		
I	-7.5	146.0	1104000		Satellite Positions
II	-27.0	156.0	1104000		Fixed by Known Stations
III	-29.5	140.0	1104000		

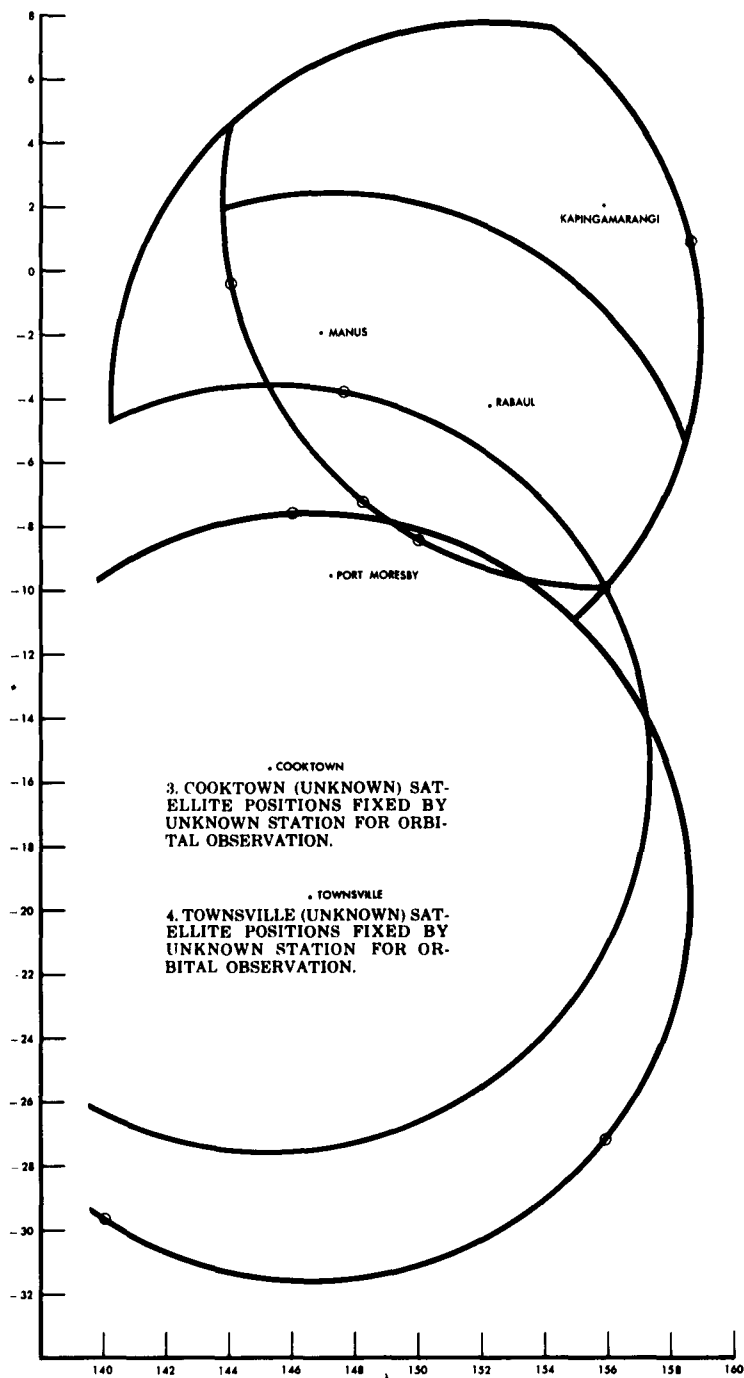
Limit 60° Zenith Distance

### FINAL RESULTS:

Port Moresby	±	209.1	Uncertainty (meters)
Cooktown	±	292.9	" "
Townsville	±	288.5	" "

SATELLITE C. PHASE 1-10  
 LIMIT 60° ZENITH DISTANCE  
 KAPINGAMARANGI, RABAU, MANUS  
 (KNOWN)  
 PORT MORESBY, COOKTOWN,  
 TOWNSVILLE  
 (UNKNOWN)

1. PORT MORESBY (UNKNOWN)  
 SATELLITE POSITIONS FIXED  
 BY KNOWN AND UNKNOWN  
 STATIONS FOR SIMULTANEOUS  
 OBSERVATIONS.
2. SATELLITE POSITIONS FIXED  
 BY KNOWN STATIONS FOR ORBI-  
 TAL OBSERVATIONS (FOR COOK-  
 TOWN AND TOWNSVILLE).



SATELLITE C

40° Inclination

Height = 1104 KM

Phase I - 11

Ulithi, Babelthwap, Kaap D'Urville, --- Catabato, Surigao  
(New Guinea)

	$\phi$	$\lambda$	h(meters)	Name	
1	10.03333	139.78333	1	Ulithi	
2	7.61667	134.58333	183	Babelthwap, (New Guinea)	Known Stations
3	-1.48333	137.9	30	Kaap D'Urville (New Guinea)	
4	7.15	124.15	683	Catabato	Unknown Station
I	-0.1	133.5	1104000		
II	6.0	128.5	1104000		Satellite Positions
III	10.3	135.5	1104000		
4	9.75	125.41667	305	Surigao	Unknown Station
I	0.2	132.8	1104000		
II	6.0	128.5	1104000		Satellite Positions
III	10.4	137.2	1104000		

Limit - 60° Zenith Distance

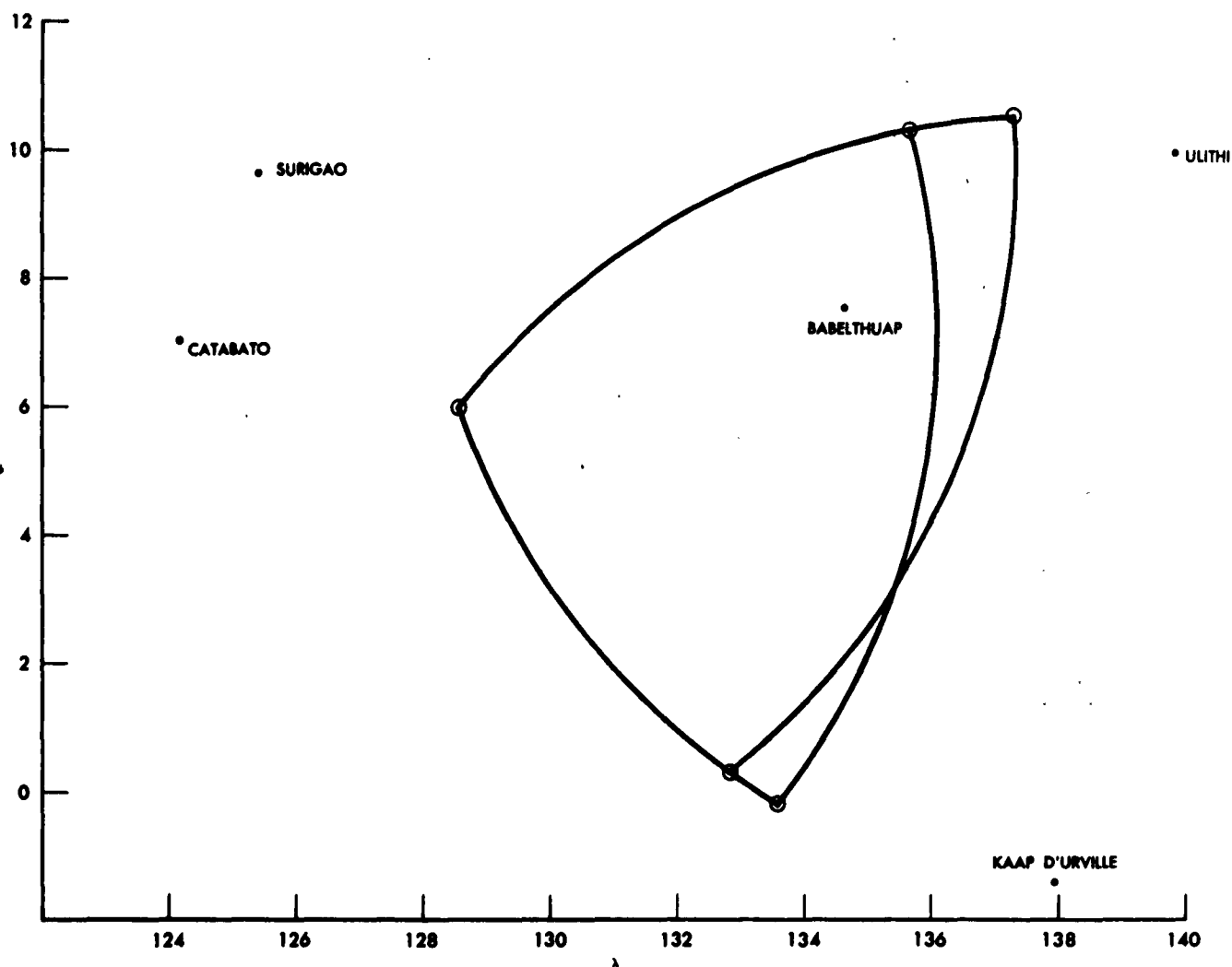
FINAL RESULTS:

Catabato  $\pm$  589.6 Uncertainty (meters)  
Surigao  $\pm$  509.5 " "



SATELLITE C, PHASE I-11  
 LIMIT 60° ZENITH DISTANCE  
 ULITHI, BABELTHUAP, KAAP D'URVILLE  
 (KNOWN)  
 CATABATO, SURIGAO  
 (UNKNOWN)

SATELLITE POSITIONS FIXED BY KNOWN  
 AND UNKNOWN STATIONS FOR SIMULTA-  
 NEOUS OBSERVATIONS.



SATELLITE B      90° Inclination  
 Height = 3200 KM

Phase I - 12

Kaap D'Urville, Kuching, Aparri --- Darwin, Wyndham

	$\phi$	$\lambda$	h(meters)	Name	
1	-1.48333	137.9	30	Kaap D'Urville	
2	1.5	110.33333	25	Kuching	Known Stations
3	18.13333	121.9	740	Aparri	
4	-12.33333	130.83333	30	Darwin	Unknown Station
I	8.4	117.2	3200 000		
II	12.5	132.7	3200 000		Satellite Positions
III	-6.8	124.0	3200 000		
4	-15.55	128.05	120	Wyndham	Unknown Station
I	6.4	116.4	3200 000		
II	8.6	134.2	3200 000		Satellite Positions
III	-6.8	124.0	3200 000		

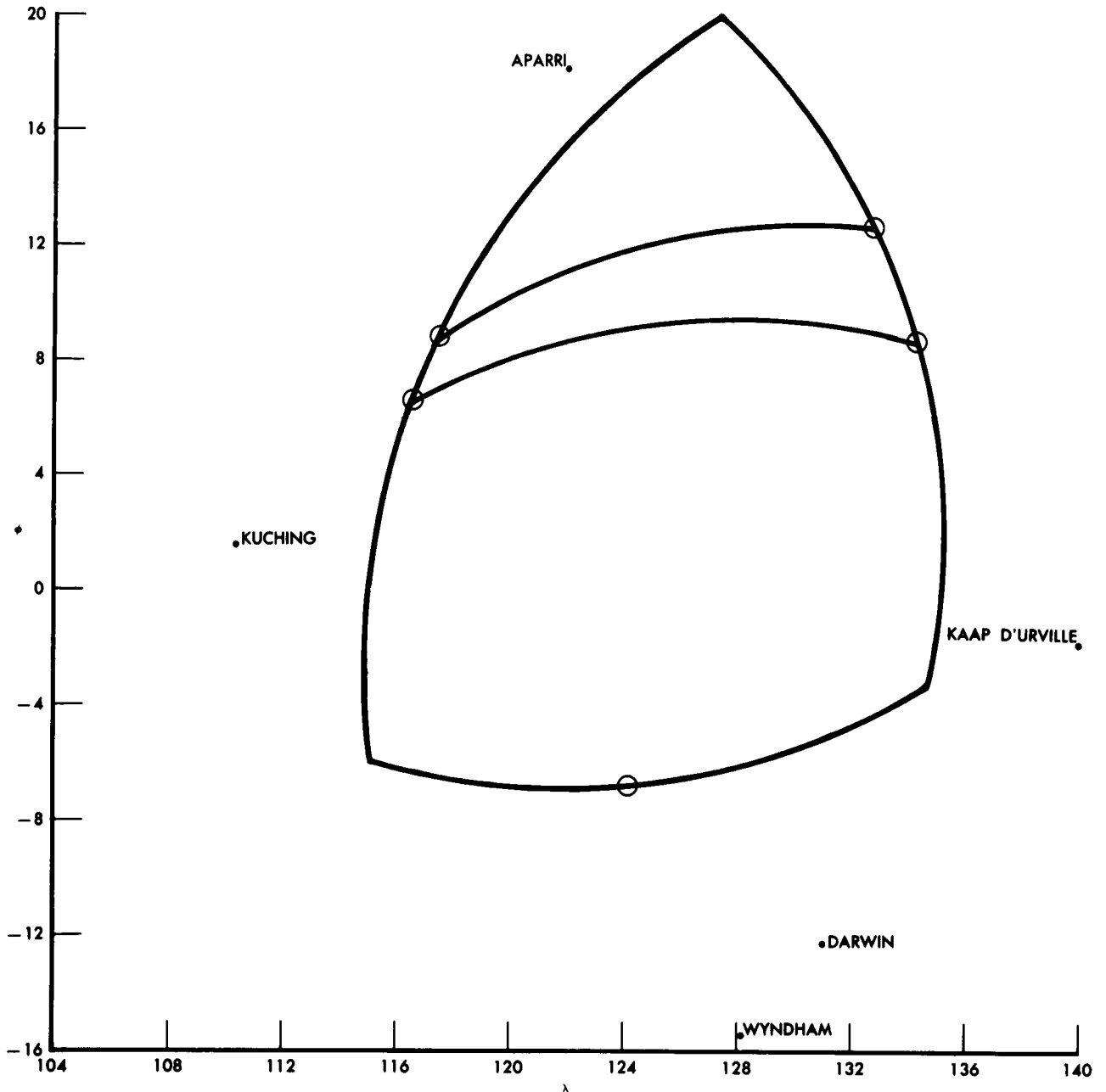
Limit - 60° Zenith Distance

FINAL RESULTS:

Darwin       $\pm$  181.6    Uncertainty (meters)  
 Wyndham     $\pm$  203.1      "            "

SATELLITE B, PHASE I-12  
 LIMIT 60° ZENITH DISTANCE  
 KAAP D'URVILLE, KUCHING, APARRI  
 (KNOWN)  
 DARWIN, WYNDHAM  
 (UNKNOWN)

STATION COORDINATES AND SATELLITE POSITIONS FIXED BY  
 THE 3 KNOWN STATIONS AND THE 2 UNKNOWN STATIONS  
 FOR THE SIMULTANEOUS OBSERVATIONS



SATELLITE C      40° Inclination  
 Height = 1104 KM

Phase II - 1

Laccadives, Maldives, Columbo - - - Cocos Island

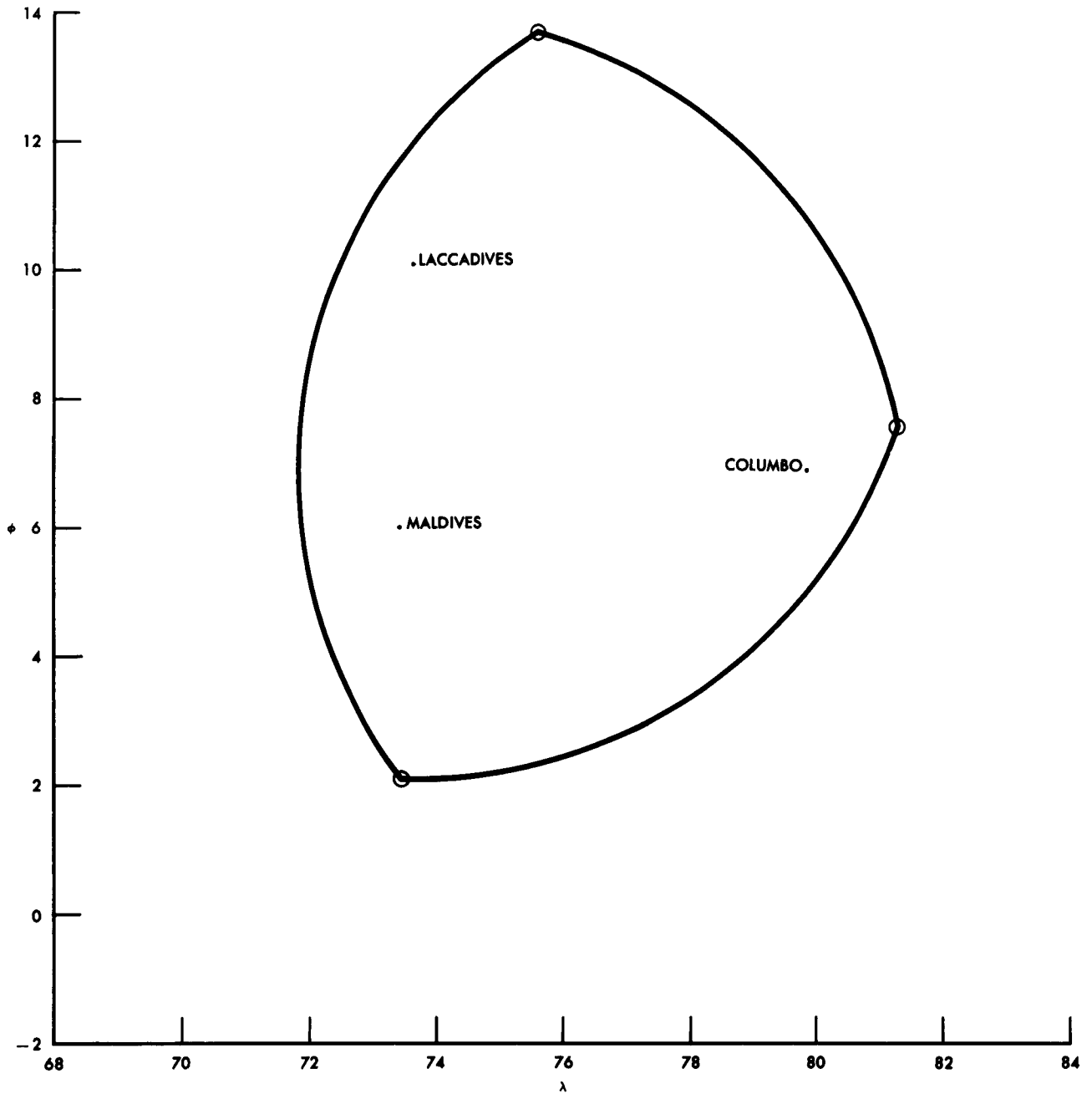
	$\phi$	$\lambda$	h(meters)	Name	
1	10.08333	73.63333	0	Laccadiv	
2	5.96667	73.38333	0	Maldives	Known Station
3	6.93333	79.85	10	Columbo	
<hr/>					
I	2.2	73.5	1104000		Satellite Positions
II	13.6	75.6	1104000		Fixed by Known Stations
III	7.6	81.0	1104000		
<hr/>					
4	-12.11667	96.91667	0	Cocos Island	Unknown Station
<hr/>					
I	-9.6	89.4	1104000		Satellite Positions
II	-13.4	104.8	1104000		Fixed by Unknown Station
III	-4.3	93.6	1104000		

Limit 60° Zenith Distance

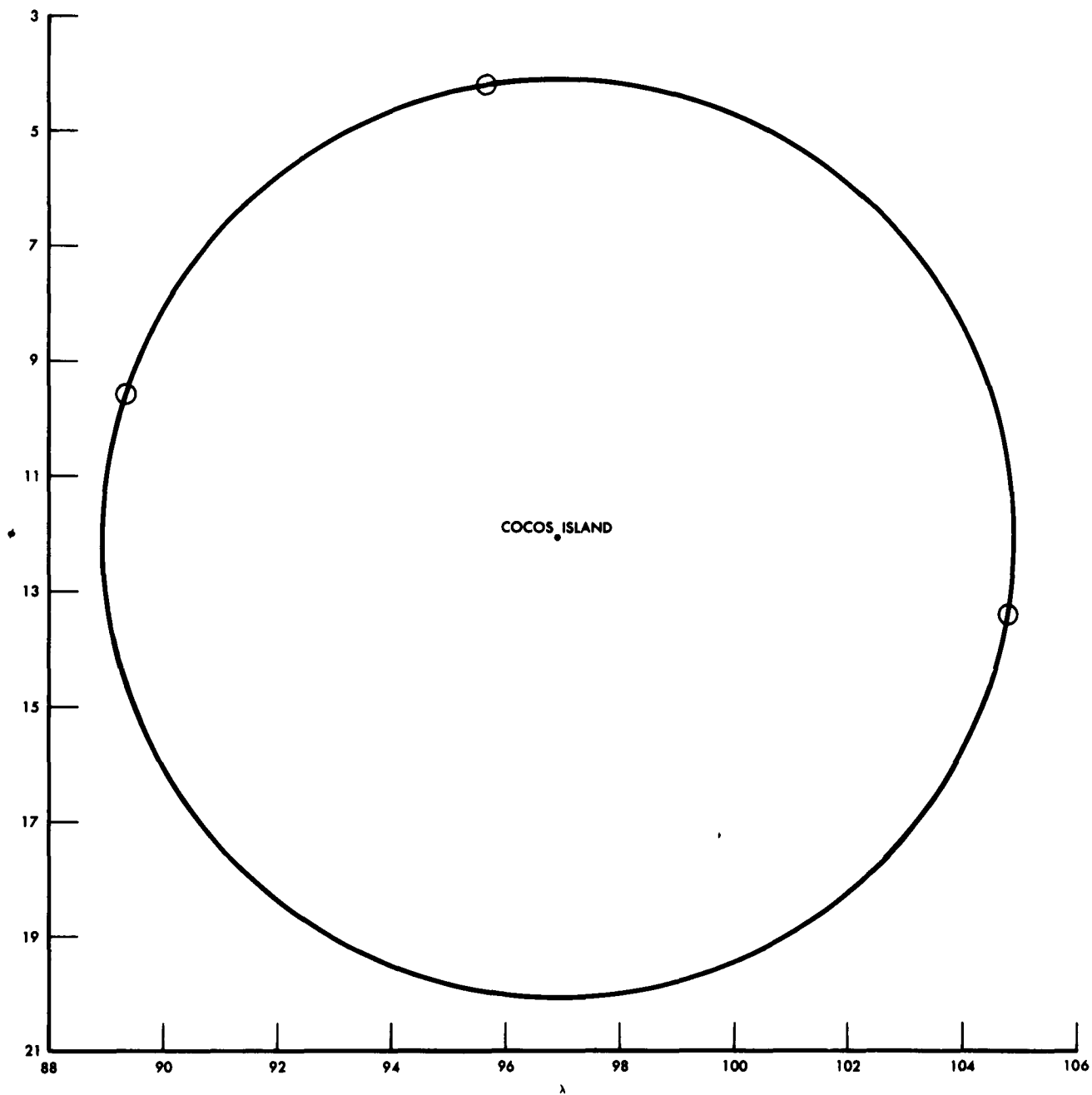
FINAL RESULTS:

Laccadives	±	89.4	Uncertainty (meters)
Maldives	±	106.8	" "
Cocos Island	±	393.5	" "

SATELLITE C, PHASE II-1  
 LIMIT 60° ZENITH DISTANCE  
 LACCADIVES, MALDIVES, CUMBO-  
 (KNOWN)  
 COCOS ISLAND  
 (UNKNOWN)  
 SATELLITE POSITIONS FIXED BY KNOWN STATIONS  
 FOR ORBITAL OBSERVATION WITH COCOS ISLAND



SATELLITE C, PHASE II-1  
 LIMIT 60° ZENITH DISTANCE  
 LACCADIVES, MALDIVES, CUMBO-  
 (KNOWN)  
 COCOS ISLAND  
 (UNKNOWN)  
 SATELLITE COORDINATES FIXED BY UNKNOWN  
 STATION FOR ORBITAL OBSERVATION



SATELLITE    B    90° Inclination  
Height = 3200 KM

Phase II - 2

Karachi, Columbo, Calcutta --- Chagos Archipelago, Dante, Socotra

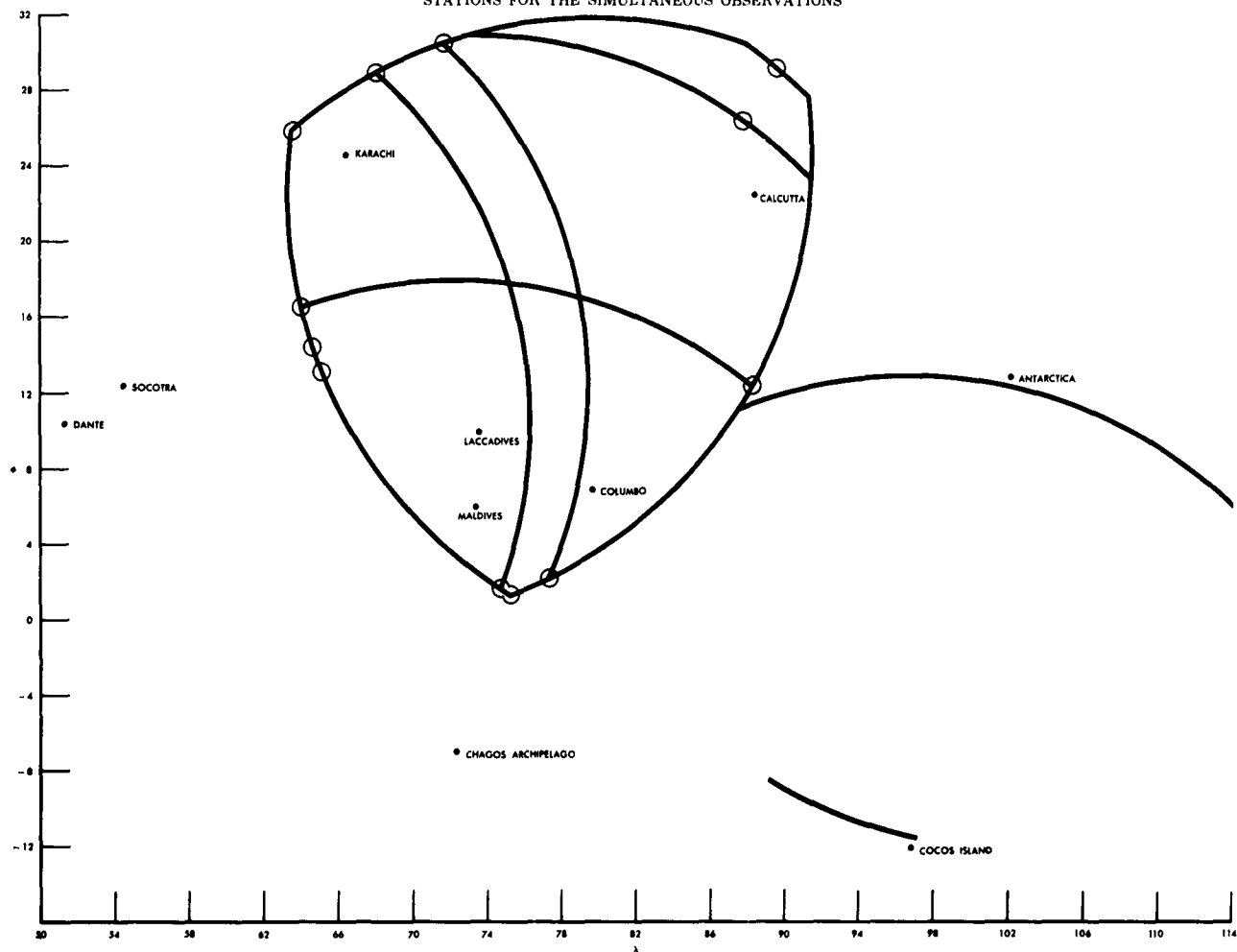
	$\phi$	$\lambda$	h(meters)	Name	
1	24.61667	66.45	25	Karachi	
2	6.93333	79.85	10	Columbo	Known Stations
3	22.55	88.35	6	Calcutta	
4	-7.31667	72.43333	2	Chagos Archipelago	Unknown Station
I	16.4	64.2	3200 000		
II	12.4	88.1	3200 000		Satellite Positions
III	1.4	75.4	3200 000		
4	10.43333	51.3	230	Dante	Unknown Station
I	13.2	65.2	3200 000		
II	28.8	68.0	3200 000		Satellite Positions
III	1.7	74.7	3200 000		
4	12.53333	54.48333	305	Socotra	Unknown Station
I	14.4	64.8	3200 000		
II	29.4	71.6	3200 000		Satellite Positions
III	2.5	77.4	3200 000		

Limit - 60° Zenith Distance

FINAL RESULTS:

Chagos Archipelago	±	215.0	Uncertainty (meters)
Dante	±	373.2	" "
Socotra	±	235.1	" "

SATELLITE B, PHASE II-2  
 LIMIT 60° ZENITH DISTANCE  
 KARACHI, COLOMBO, CALCUTTA-  
 (KNOWN)  
 CHAGOS ARCHIPELAGO, DANTE, SOCOTRA  
 (UNKNOWN)  
 STATION COORDINATES AND SATELLITE POSITIONS FIXED  
 BY THE THREE KNOWN STATIONS AND THE TWO UNKNOWN  
 STATIONS FOR THE SIMULTANEOUS OBSERVATIONS





90° Inclination  
SATELLITE B  
Height = 3200 KM

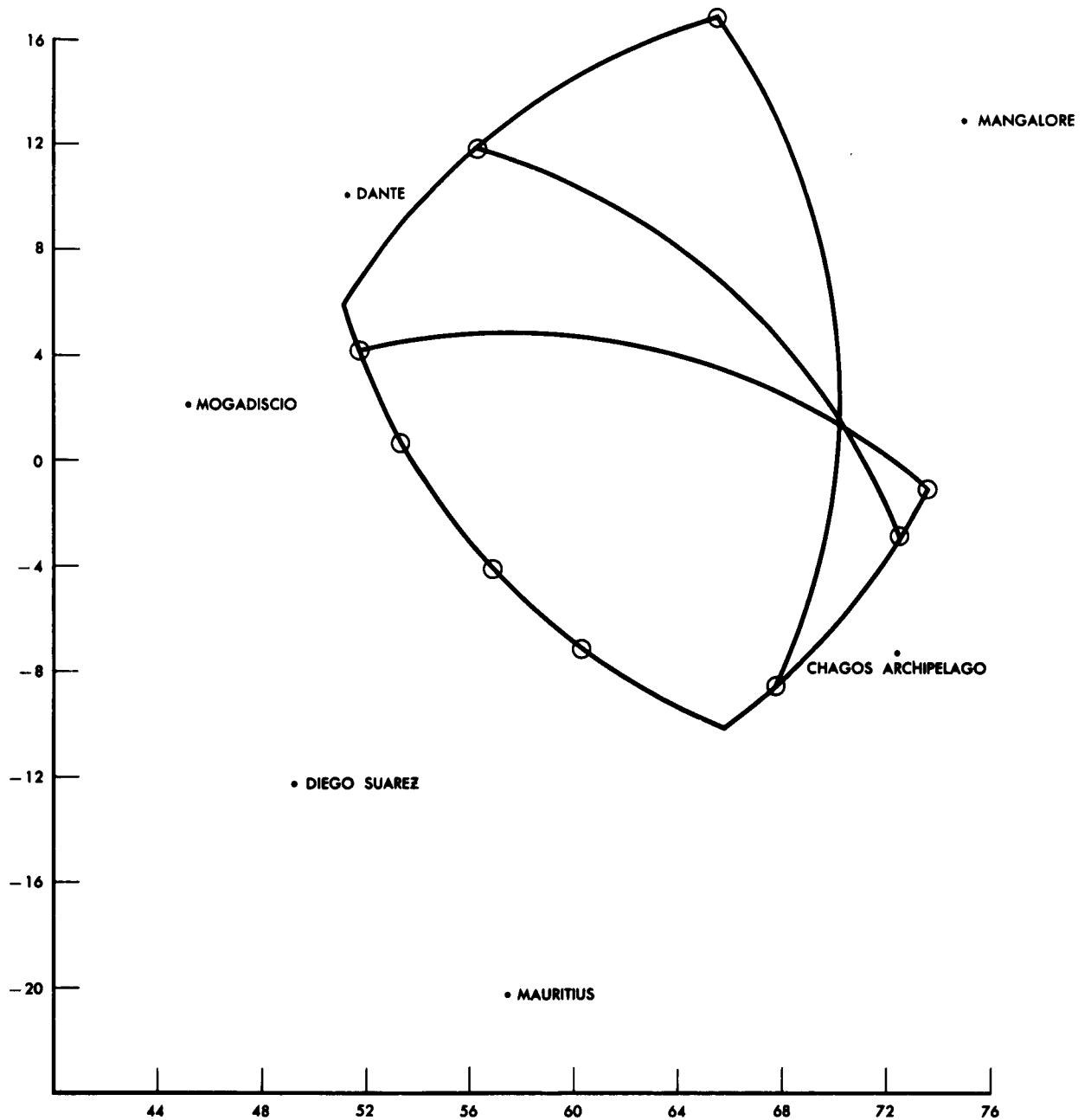
Phase II - 3

Dante, Chagos Archipelago, Mangalore --- Mogadiscio, Diego Suarez, Mauritius					
	$\phi$	$\lambda$	h(meters)	<u>Name</u>	
1	10.43333	51.3	230	Dante	Known Stations
2	-7.31667	72.43333	2	Chagos Archipelago	
3	12.83333	75.0	31	Mangalore	
4	2.03333	45.33333	13	Mogadiscio	Unknown Station
I	16.6	65.4	3200 000		
II	0.8	53.1	3200 000		Satellite Positions
III	-8.6	68.0	3200 000		
4	-12.26667	49.3	4	Diego Suarez	Unknown Station
I	11.6	56.3	3200 000		
II	-4.0	56.6	3200 000		Satellite Positions
III	-3.2	72.5	3200 000		
4	-20.16667	57.48333	305	Mauritius	Unknown Station
I	4.0	51.6	3200 000		
II	-7.2	60.0	3200 000		Satellite Positions
III	-1.1	73.6	3200 000		

FINAL RESULTS:

Mogadiscio	$\pm$	155.7	Uncertainty (meters)	
Diego Suarez	$\pm$	248.9	"	"
Mauritius	$\pm$	309.7	"	"

SATELLITE B, PHASE II-3  
 LIMIT 60° ZENITH DISTANCE  
 DANTE, CHAGOS ARCHIPELAGO, MANGALORE  
 (KNOWN)  
 MOGADISCIO, DIEGOSUAREZ, MAURITIUS  
 (UNKNOWN)



90° Inclination  
 SATELLITE B  
 Height = 3200 KM

Phase II - 4

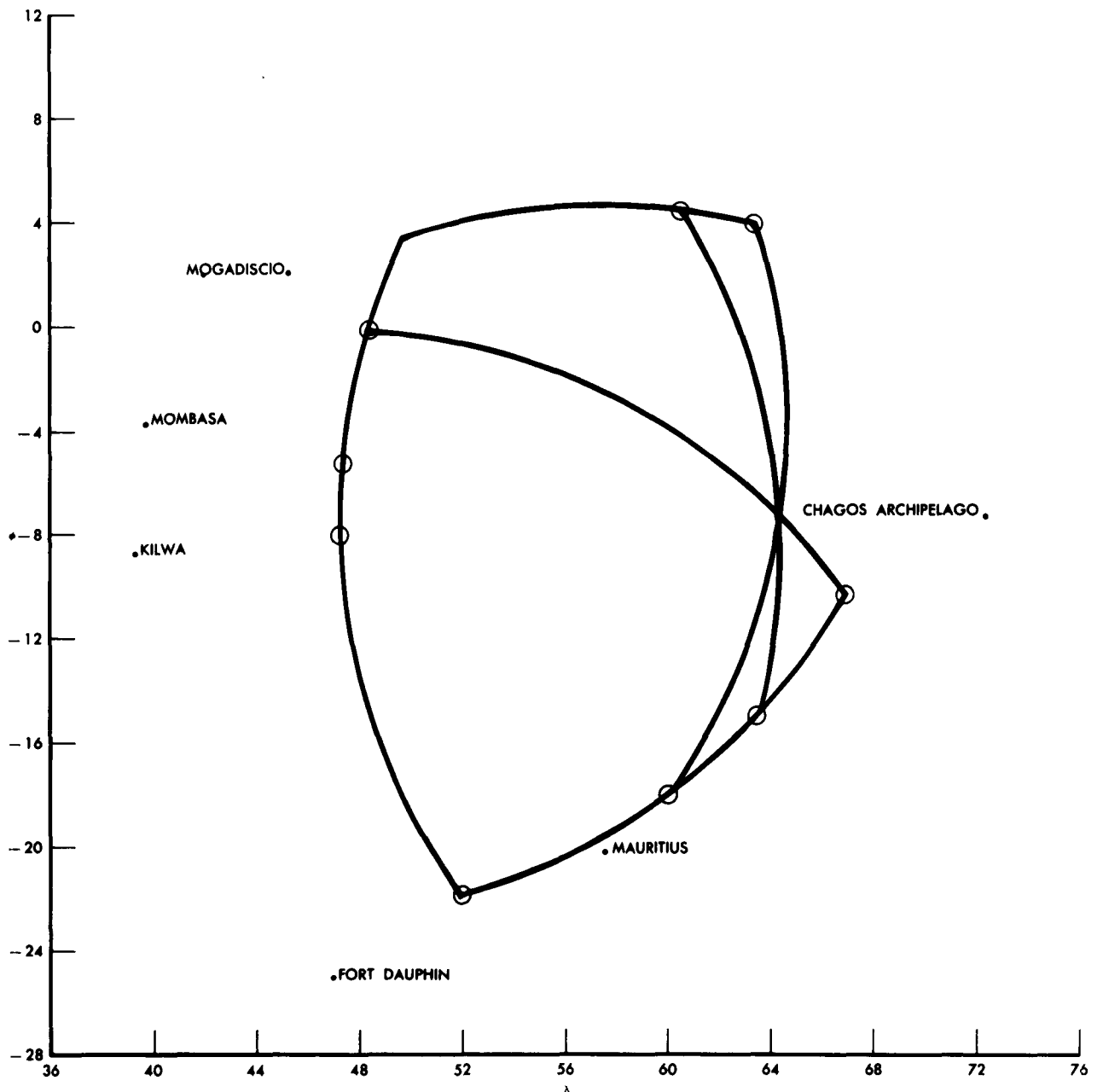
Chagos Archipelago, Mogadiscio, Mauritius -- Mombasa, Ft. Dauphin, Kilwa					
	$\phi$	$\lambda$	h(meters)	<u>Name</u>	
1	-7.31667	72.43333	2	Chagos Archipelago	
2	2.03333	45.33333	13	Mogadiscio	Known Stations
3	-20.16667	57.48333	305	Mauritius	
4	-4.06667	39.68333	5	Mombasa	Unknown Station
I	-5.6	47.4	3200 000		
II	4.0	63.2	3200 000		Satellite Positions
III	-18.0	60.0	3200 000		
4	-25.03333	47.0	1000	Ft. Dauphin	Unknown Station
I	-21.6	52.2	3200 000		
II	-0.3	48.6	3200 000		Satellite Positions
III	-10.2	67.0	3200 000		
4	-8.75	39.25	100	Kilwa	Unknown Station
I	-8.0	47.4	3200 000		
II	4.5	60.4	3200 000		Satellite Positions
III	-15.0	63.4	3200 000		

FINAL RESULTS:

Mombasa	$\pm$	156.8	Uncertainty (meters)
Fort Dauphin	$\pm$	173.1	" "
Kilwa	$\pm$	157.6	" "

SATELLITE B PHASE II-4  
 LIMIT 60° ZENITH DISTANCE  
 CHAGOS ARCHIPELAGO, MOGADISCIO, MAURITIUS  
 (KNOWN)  
 MOMBASA, FORT DAUPHIN, KILWA  
 (UNKNOWN)

STATION COORDINATES AND SATELLITE POSITIONS FIXED  
 BY THE 3 KNOWN AND 3 UNKNOWN STATIONS FOR THE  
 SIMULTANEOUS OBSERVATIONS



SATELLITE B      90° Inclination  
                          Height = 3200 KM

Phase II - 5 & 6

Lusaka, Brazzaville, Luderitz - - - St. Helena

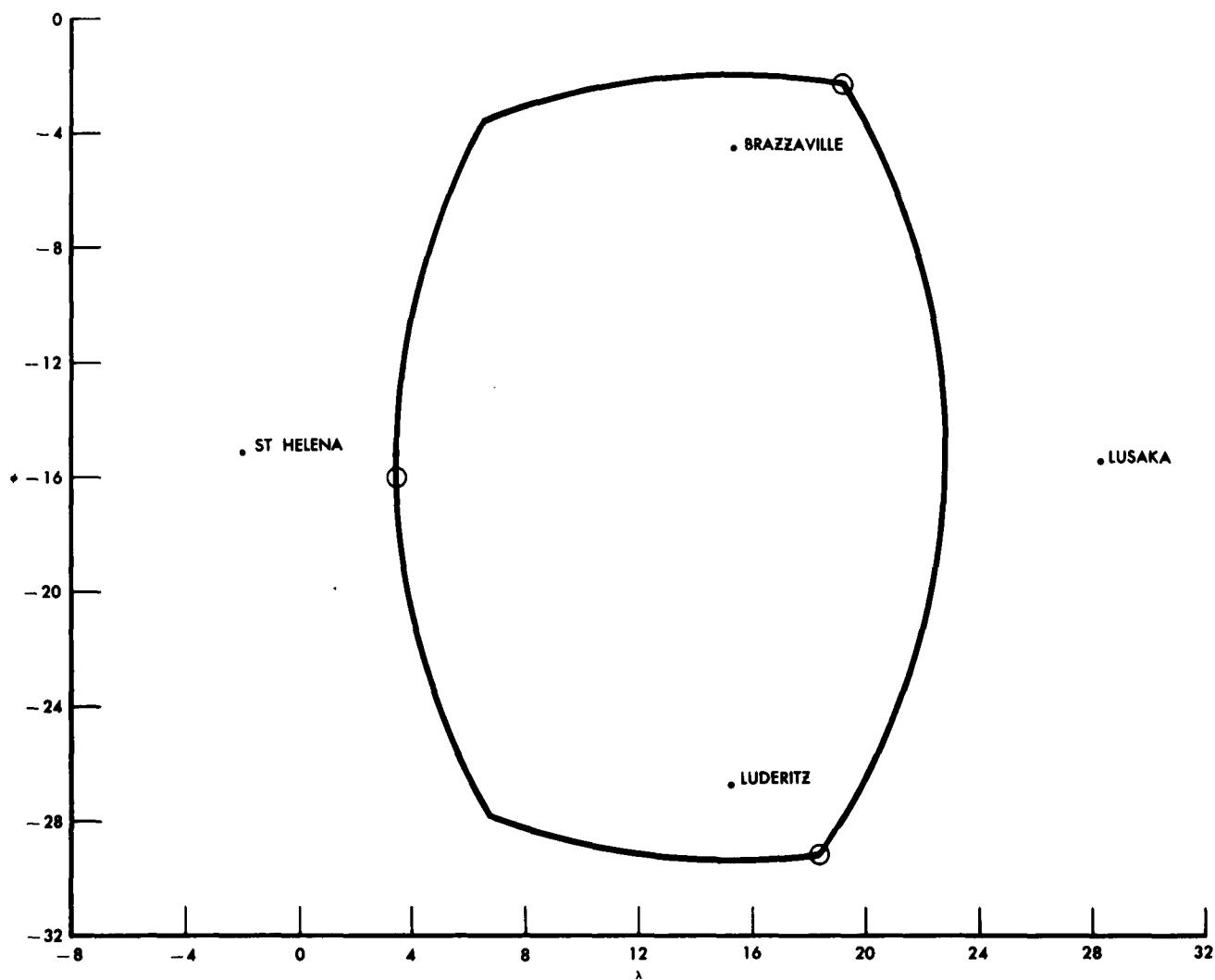
	$\phi$	$\lambda$	h(meters)	<u>Name</u>	
1	-15.41667	28.3	5	Lusaka	
2	-4.51667	15.28333	318	Brazzaville	Known Stations
3	-26.65	15.15	5	Luderitz	
4	-15.91667	-5.91667	305	St. Helena	Unknown Station
I	-16.0	3.4	3200 000		
II	-2.2	19.2	3200 000		Satellite Positions
III	-29.1	18.6	3200 000		

FINAL RESULTS:

Brazzaville     $\pm$  267.8    Uncertainty (meters)  
 St. Helena     $\pm$  231.0    "            "

SATELLITE B, PHASE II-5, 6  
 LIMIT 60° ZENITH DISTANCE  
 LUSAKA, BRAZZAVILLE, LUDERITZ  
 (KNOWN)  
 ST HELENA  
 (UNKNOWN)

STATION COORDINATES AND SATELLITE POSITIONS  
 FIXED BY THE 3 KNOWN AND 1 UNKNOWN STATION  
 FOR THE SIMULTANEOUS OBSERVATION



90° Inclination  
 SATELLITE 1  
 Height = 3200 KM

Phase II - 7

Brazzaville, St. Helena, Walvisbaa --- Ascension, Accra, Lagos

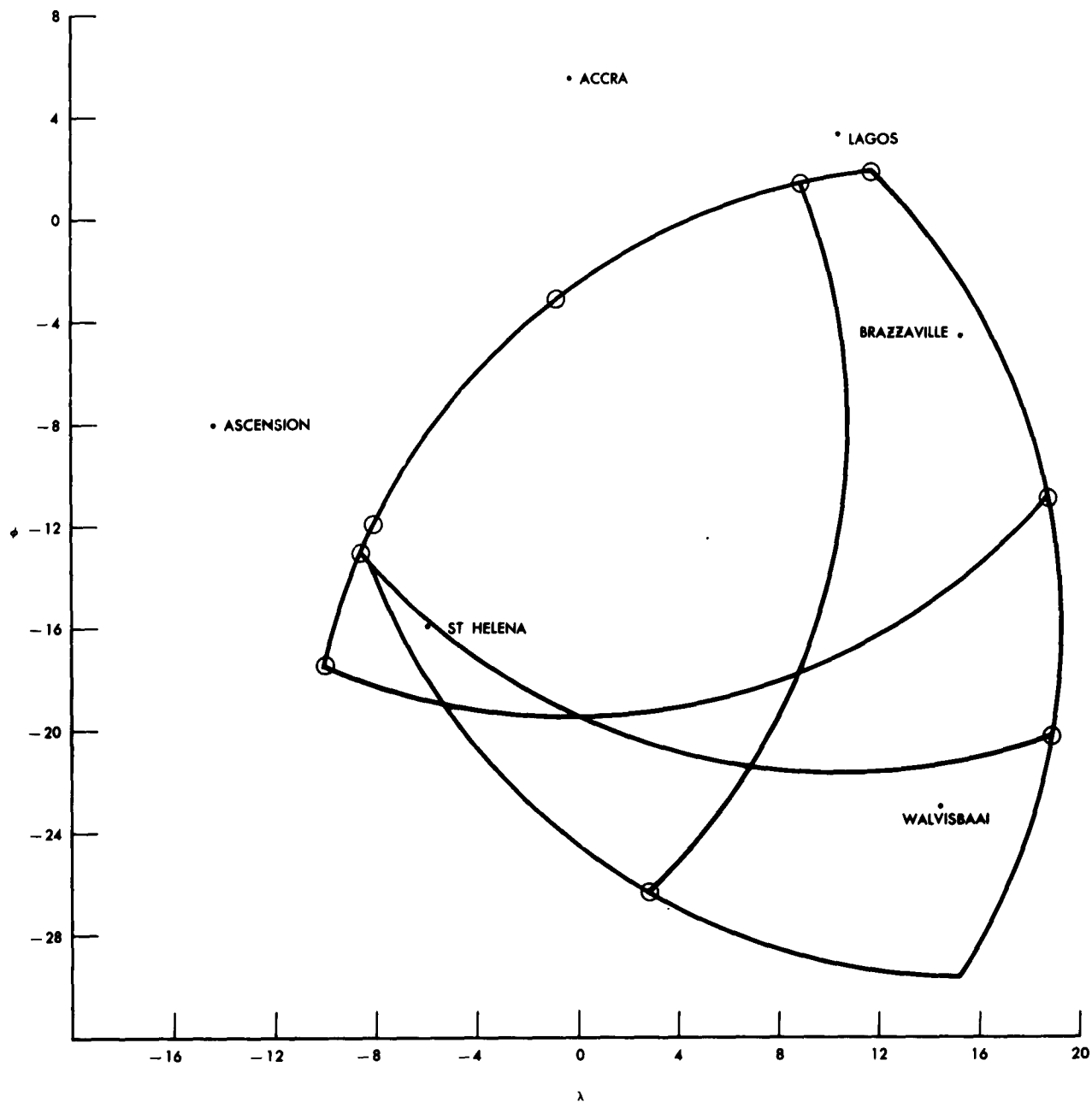
	$\phi$	$\lambda$	h(meters)	<u>Name</u>	
1	-4.51667	15.28333	318	Brazzaville	
2	-15.91667	-5.91667	305	St. Helena	Known Stations
3	-22.96667	14.5	5	Walvisbaa	
4	-7.95	-14.36667	287	Ascension	Unknown Station
I	-26.0	6.9	3200 000		
II	-12.0	-8.0	3200 000		Satellite Positions
III	1.2	4.6	3200 000		
4	5.58333	-0.25	67	Accra	Unknown Station
I	-17.6	-9.0	3200 000		
II	-3.2	-0.8	3200 000		Satellite Positions
III	-10.8	18.4	3200 000		
4	6.45	3.41667	40	Lagos	Unknown Station
I	-12.6	-8.4	3200 000		
II	1.5	11.6	3200 000		Satellite Positions
III	-20.0	18.7	3200 000		

LIMIT - 60° Zenith Distance

**FINAL RESULTS:**

Ascension	$\pm$	179.4	Uncertainty (meters)	
Accra	$\pm$	204.9	"	"
Lagos	$\pm$	180.4	"	"

SATELLITE B, PHASE II-7  
 LIMIT 60° ZENITH DISTANCE  
 BRAZZAVILLE, ST HELENA, WALVISBAAI  
 (KNOWN)  
 ASCENSION, ACCRA, LAGOS  
 (UNKNOWN)  
 STATION COORDINATES AND SATELLITE POSITIONS  
 FIXED BY THE 3 KNOWN AND 3 UNKNOWN STATIONS  
 FOR THE SIMULTANEOUS OBSERVATIONS





90° Inclination  
 SATELLITE B  
 HEIGHT = 3200 KM

Phase II - 8

Ascension, Brazzaville, Accra --- Freetown, Dakar, Boa Vista

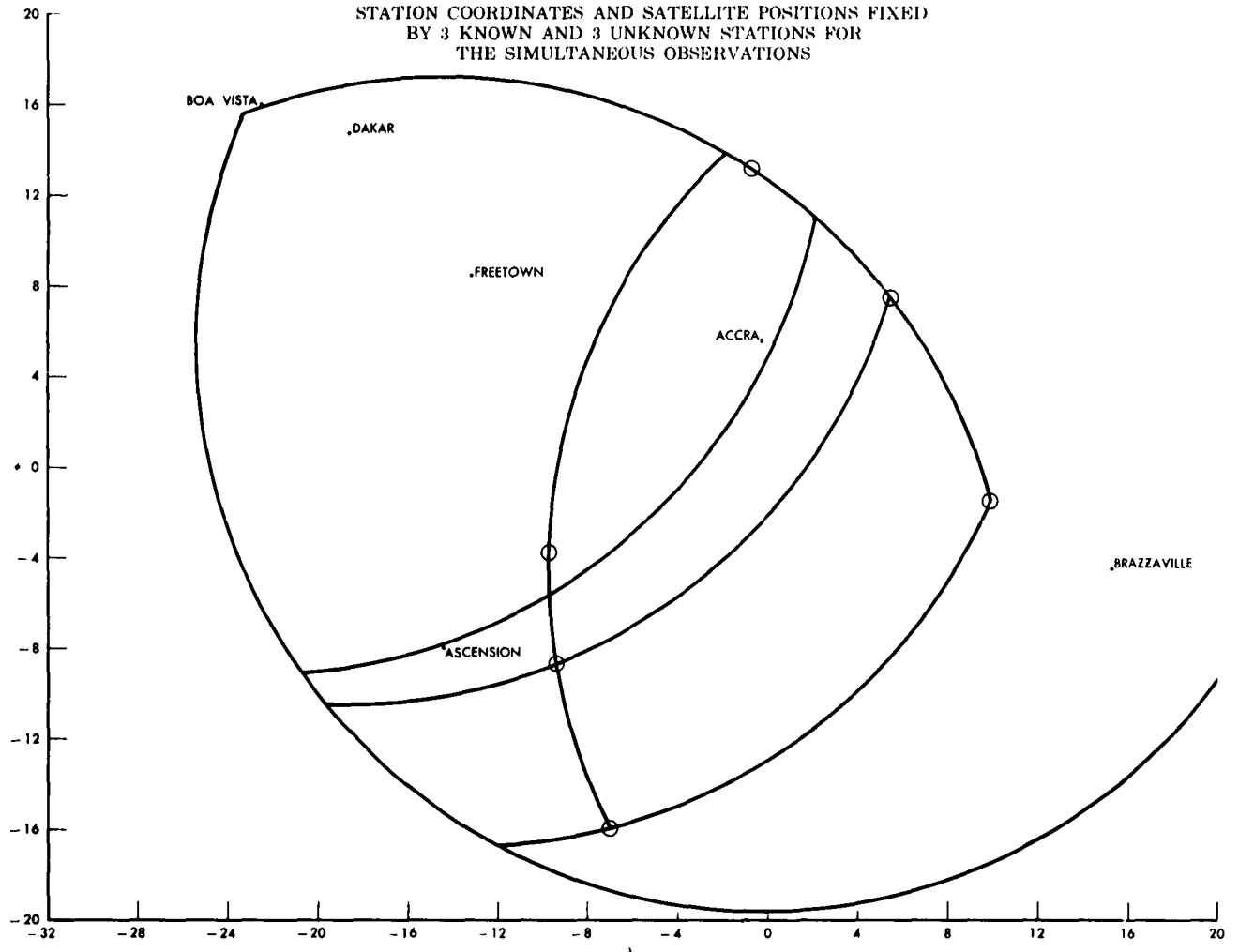
	$\phi$	$\lambda$	h(meters)	<u>Name</u>	
1	-7.95	-14.36667	287	Ascension	
2	-4.51667	15.28333	318	Brazzaville	Known Stations
3	5.583333	0.25	67	Accra	
4	8.48333	-13.23333	305	Freetown	Unknown Station
I	6.8	-7.2	3200 000		
II	-15.8	-6.9	3200 000		Satellite Positions
III	-1.3	9.7	3200 000		
4	14.66667	-18.55	23	Dakar	Unknown Station
I	6.8	-7.2	3200 000		
II	-8.7	-9.2	3200 000		Satellite Positions
III	7.4	5.2	3200 000		
4	16.01667	-22.55	31	Boa Vista	Unknown Station
I	6.8	-7.2	3200 000		
II	-3.5	-9.6	3200 000		Satellite Positions
III	13.0	-0.8	3200 000		

FINAL RESULTS:

Freetown	$\pm$	148.7	Uncertainty (meters)	
Dakar	$\pm$	158.5	"	"
Boa Vista	$\pm$	202.1	"	"

SATELLITE B, PHASE II-8  
 LIMIT 60° ZENITH DISTANCE  
 ASCENSION, BRAZZAVILLE, ACCRA  
 (KNOWN)  
 FREETOWN, BOA VISTA, DAKAR  
 (UNKNOWN)

STATION COORDINATES AND SATELLITE POSITIONS FIXED  
 BY 3 KNOWN AND 3 UNKNOWN STATIONS FOR  
 THE SIMULTANEOUS OBSERVATIONS



SATELLITE B 90° Inclination

Height = 3200 KM

Phase II - 9

Accra, Brazzaville, Ascension -- Tetuan, Oran

	$\phi$	$\lambda$	h(meters)	Name	
1	5.58333	-0.25	67	Accra	
2	-4.51667	15.28333	318	Brazzaville	Known Stations
3	-7.95	-14.36667	287	Ascension	
I	-18.8	-5.0	3200 000		Satellite Positions Fixed by Known Stations
II	13.9	-2.0	3200 000		
III	-17.8	8.4	3200 000		
4	35.58333	-5.38333	10	Tetuan	Unknown Station
I	60.2	-1.6	3200 000		Satellite Positions Fixed by Unknown Station
II	13.5	-17.2	3200 000		
III	28.8	18.4	3200 000		
4	35.33333	-0.3	31	Oran	Unknown Station
I	60.2	-1.6	3200 000		Satellite Positions Fixed by Unknown Station
II	17.0	-17.2	3200 000		
III	18.8	18.4	3200 000		

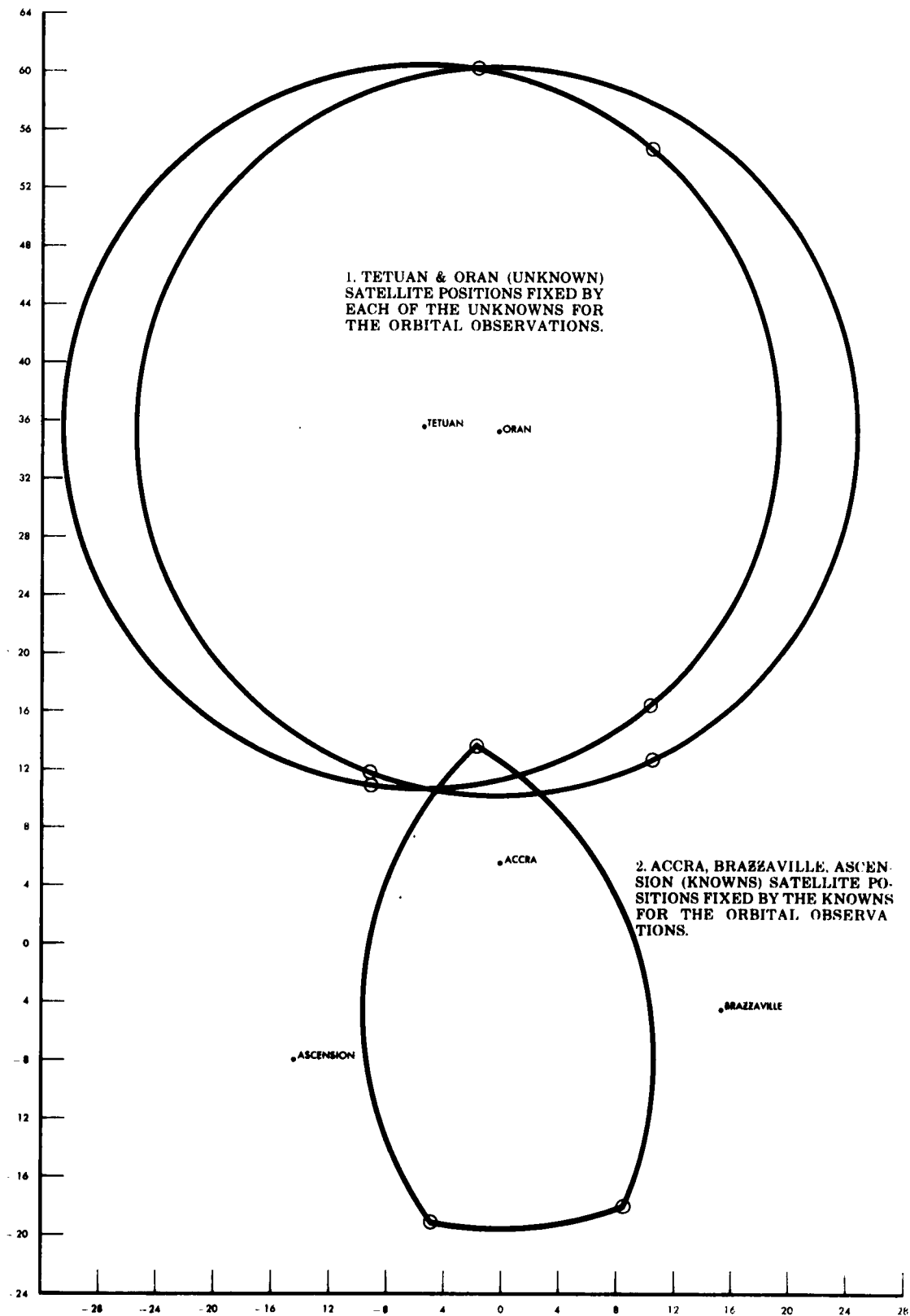
Limit 60° Zenith Distance

FINAL RESULTS:

Tetuan  $\pm 234.6$  Uncertainty (meters)

Oran  $\pm 199.4$  " "

SATELLITE B, PHASE II-9  
 LIMIT 60° ZENITH DISTANCE  
 ACCRA, BRAZZAVILLE, ASCENSION  
 (KNOWN)  
 TETUAN, ORAN  
 (UNKNOWN)



SATELLITE B      90° Inclination  
Height = 3200 KM

Phase III - 1

Ascension, Accra, Dakar --- Fernando de Noronha, Recife

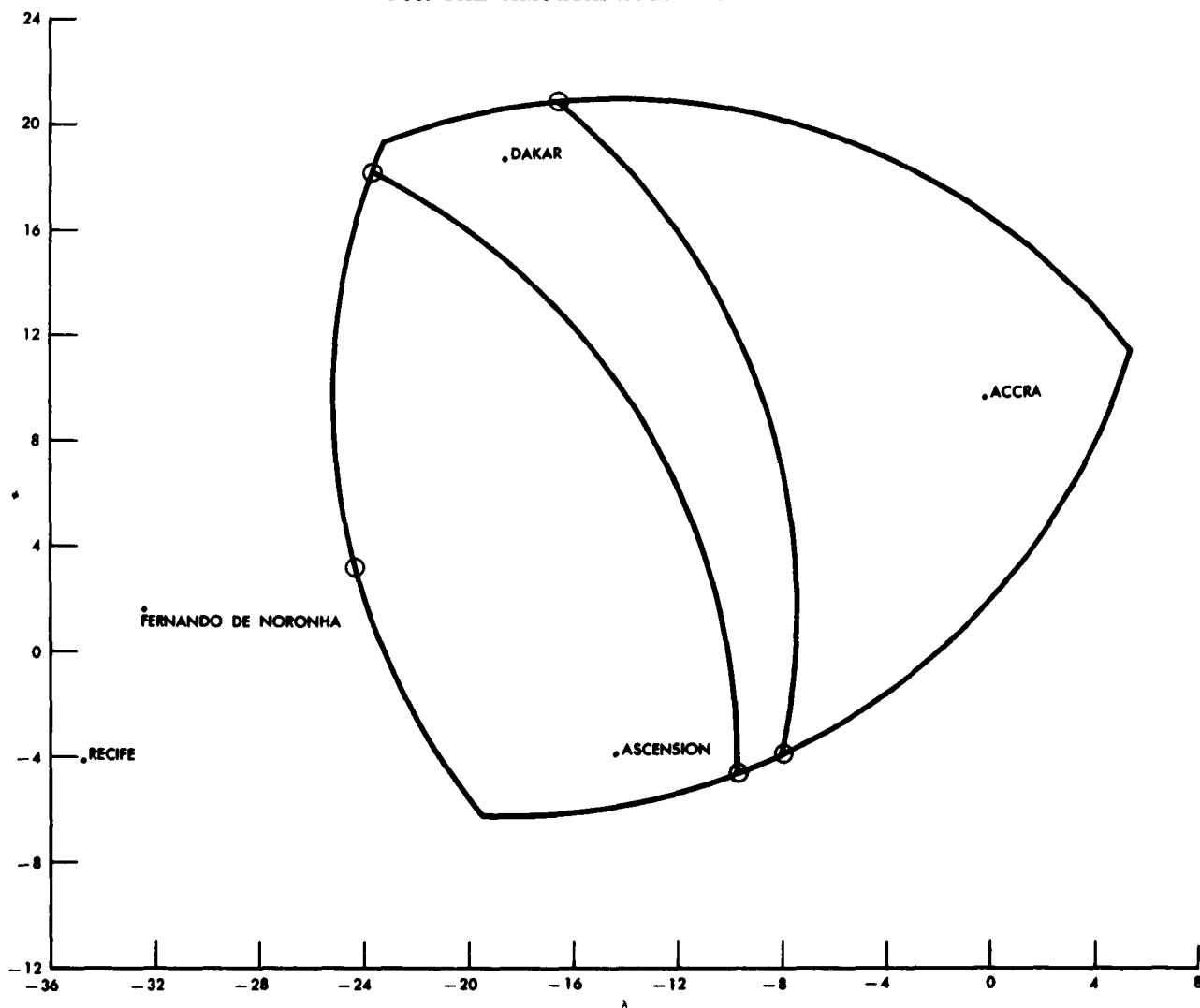
	$\phi$	$\lambda$	h(meters)	<u>Name</u>	
1	-7.95	-14.36667	287	Ascension	
2	5.58333	-0.25	67	Accra	Known Stations
3	14.66667	-18.55	23	Dakar	
4	-3.66667	-32.41667	50	Fernando de Noronha	Unknown Station
I	-0.8	-24.4	3200 000		
II	16.8	-16.7	3200 000		Satellite Positions
III	-7.8	-8.0	3200 000		
4	-8.15	-34.78333	100	Recife	Unknown Station
I	-0.8	-24.4	3200 000		
II	14.1	-23.7	3200 000		Satellite Positions
III	-8.6	-9.8	3200 000		

Limit - 60° Zenith Distance

FINAL RESULTS:

Fernando de Noronha	±	200.5	Uncertainty (meters)
Recife	±	312.5	" "

SATELLITE B, PHASE III-1  
 LIMIT 60° ZENITH DISTANCE  
 ASCENSION, ACCRA, DAKAR  
 (KNOWN)  
 FERNANDO DE NORONHA, RECIFE  
 (UNKNOWN)  
 STATION COORDINATES AND SATELLITE POSITIONS  
 FIXED BY THE 3 KNOWN AND 2 UNKNOWN STATIONS  
 FOR THE SIMULTANEOUS OBSERVATIONS



SATELLITE C      40° Inclination  
 Height = 1104 KM

Phase III - 2

Fernando de Noronha, Parnaiba, Aracaju --- Freetown

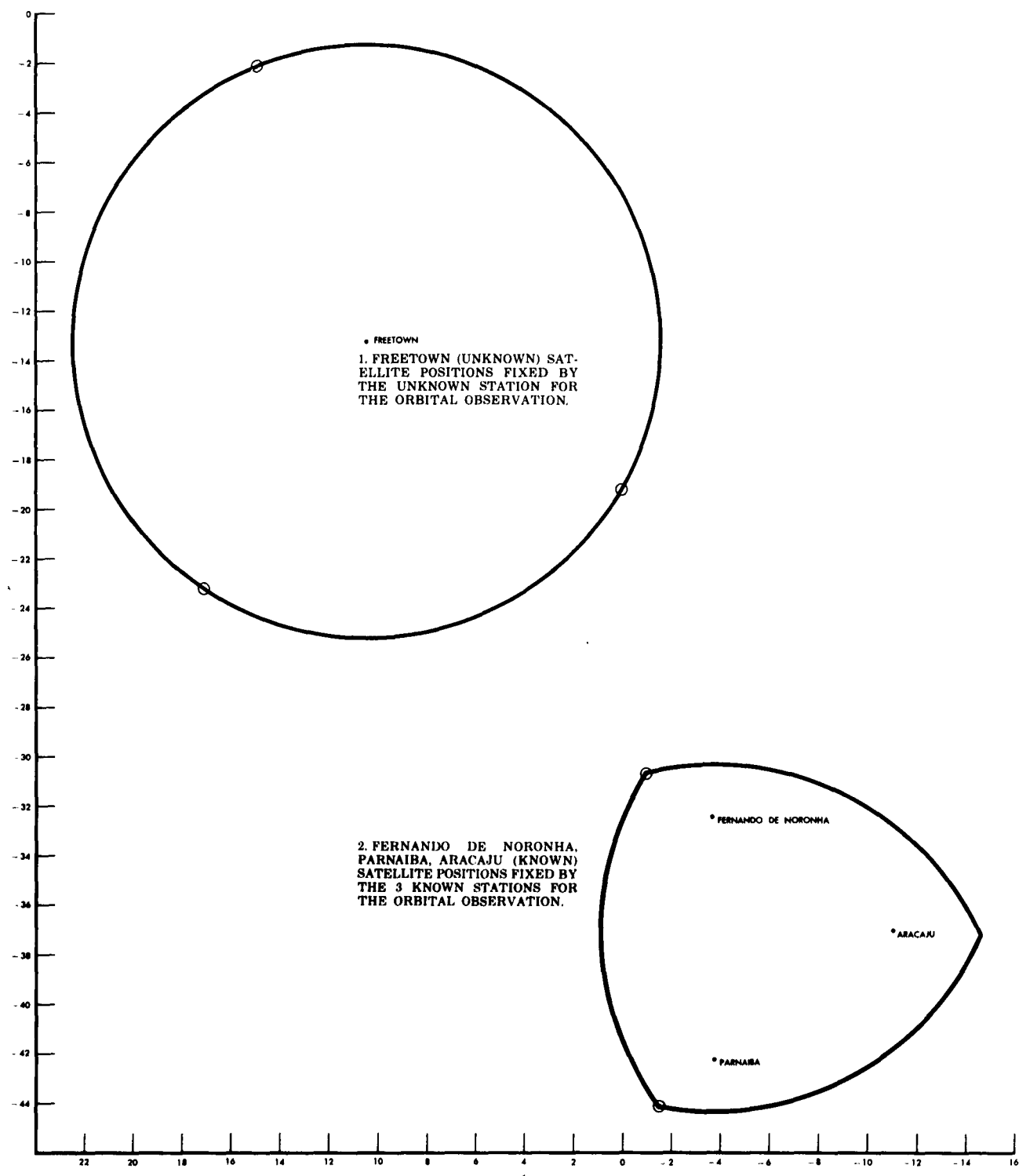
	$\phi$	$\lambda$	h(meters)	Name	
1	-3.66667	-32.41667	50	Fernando de Noronha	
2	-3.75	-42.25	50	Parnaiba	Known Stations
3	-11.03333	-37.03333	100	Aracaju	
I	-1.5	-44.1	1104 000		Satellite Positions
II	-0.8	-30.7	1104 000		Fixed by Known Stations
III	-14.6	-38.2	1104 000		
4	8.5	-13.16667	305	Freetown	Unknown Station
I	0.1	-20.1	1104 000		Satellite Positions
II	17.1	-30.2	1104 000		Fixed by Unknown Station
III	15.0	-2.2	1104 000		

Limit - 60° Zenith Distance

FINAL RESULTS:

Freetown  $\pm$  151.1 Uncertainty (meters)

SATELLITE C, PHASE III-2  
 FERNANDO DE NORONHA, PARNAIBA, ARACAJU  
 (KNOWN)  
 FREETOWN  
 (UNKNOWN)





90° Inclination  
 SATELLITE D Height = 3200 KM

Phase III - 3

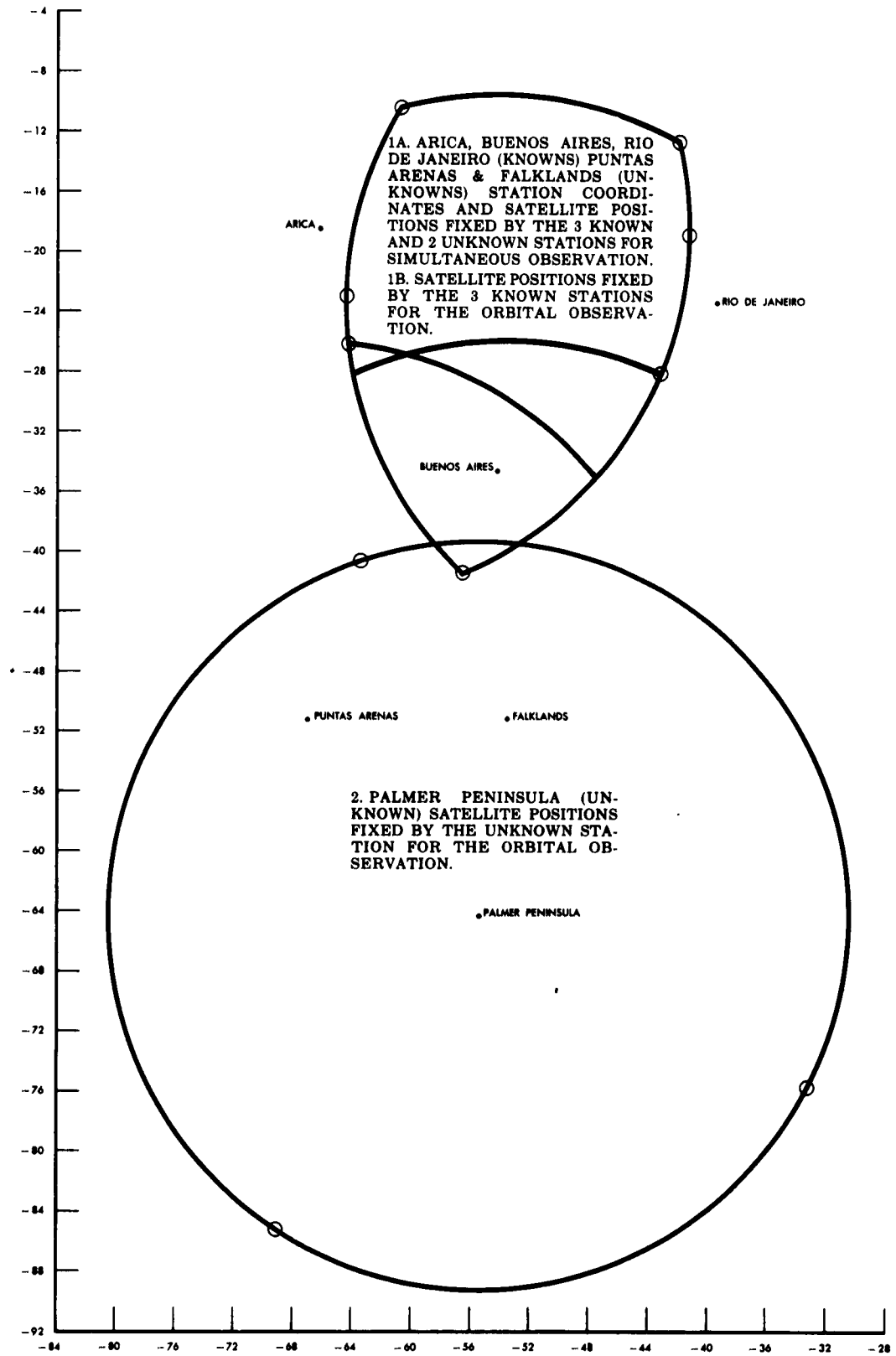
Arica, Buenos Aires, Rio de Janeiro -- Palmer Peninsula, Punta Arenas  
 Falkland Islands

	$\phi$	$\lambda$	h(meters)	Name	
1	-18.53333	-70.36667	300	Arica	
2	-34.58333	-58.33333	25	Buenos Aires	Known Stations
3	-23.25	-43.5	300	Rio de Janeiro	
4	-64.25	-59.5	100	Palmer Peninsula	Unknown Station
I	-25.8	-68.4	3200 000		Satellite Positions Fixed by Known Stations
II	-28.2	-47.4	3200 000		
III	-41.5	-60.7	3200 000		
I	-40.8	-67.6	3200 000		Satellite Positions Fixed by Unknown Station
II	-76.0	-37.2	3200 000		
III	-85.2	-73.2	3200 000		
4	-53.16667	-71.0	400	Punta Arenas	Unknown Station
I	-25.8	-68.4	3200 000		
II	-35.0	-51.7	3200 000		Satellite Positions
III	-41.6	-60.6	3200 000		
4	-51.08333	-57.65	200	Falkland Islands	Unknown Station
I	-28.2	-68.0	3200 000		
II	-28.2	-47.5	3200 000		Satellite Positions
III	-41.6	-60.6	3200 000		

FINAL RESULTS:

Palmer Peninsula	±	435.5	Uncertainty (meters)
Punta Arenas	±	410.7	" "
Falkland Islands	±	261.9	" "

SATELLITE D, PHASE III-3  
 LIMIT 60° ZENITH DISTANCE  
 ARICA, BUENOS AIRES, RIO DE JANEIRO  
 (KNOWN)  
 PALMER PENINSULA, PUNTAS ARENAS, FALKLANDS  
 (UNKNOWN)



90° Inclination  
 SATELLITE D  
 Height = 3200 KM

Phase III - 4

Parnaiba, Rio de Janeiro, Arica -- Trinidad, Buenos Aires

	$\phi$	$\lambda$	h(meters)	Name	
1	-3.75	-42.25	50	Parnaiba	
2	-23.25	-43.5	300	Rio de Janeiro	Known Stations
3	-18.53333	-70.36667	300	Arica	
4	10.5	-61.5	200	Trinidad	Unknown Station
I	-14.2	-64.8	3200 000		
II	-4.0	-41.3	3200 000		Satellite Positions
III	-2.8	-57.6	3200 000		
4	-34.58333	-58.33333	25	Buenos Aires	Unknown Station
I	-10.8	-65.5	3200 000		
II	-21.8	-36.9	3200 000		Satellite Positions
III	-26.0	-53.2	3200 000		

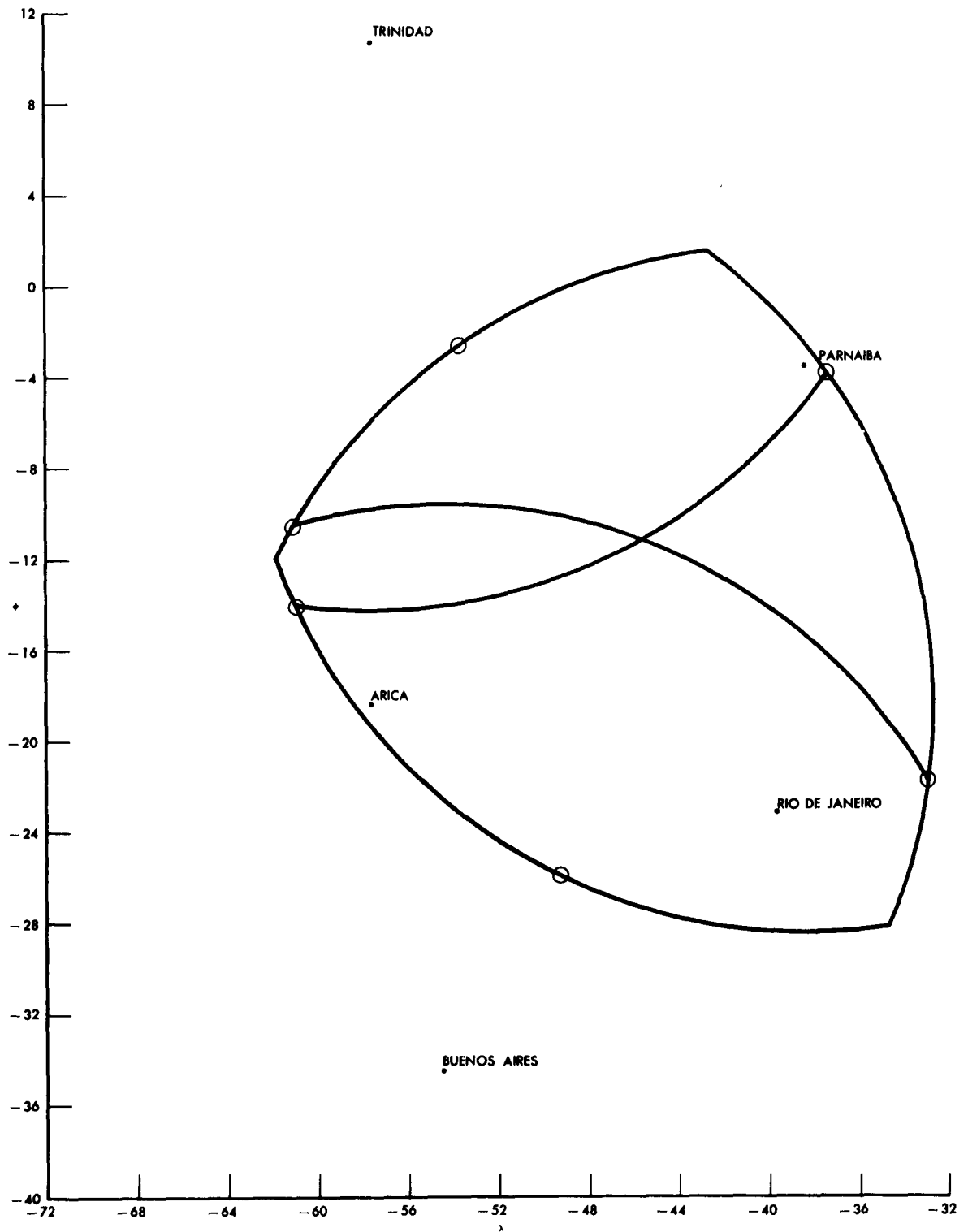
FINAL RESULTS:

Trinidad  $\pm$  265.6 Uncertainty (meters)

Buenos Aires  $\pm$  194.6 " "

SATELLITE D, PHASE III-4  
 LIMIT 60° ZENITH DISTANCE  
 PARNAIBA, RIO DE JANEIRO, ARICA  
 (KNOWN)  
 TRINIDAD, BUENOS AIRES  
 (UNKNOWN)

STATION COORDINATES AND SATELLITE POSITIONS  
 FIXED BY THE 3 KNOWN AND 2 UNKNOWN STATIONS  
 FOR THE SIMULTANEOUS OBSERVATIONS



SATELLITE D      90° Inclination  
 Height = 3200 KM

Phase III - 5

Trinidad, San Jose, Lima --- Galapagos

	$\phi$	$\lambda$	h(meters)	<u>Name</u>	
1	10.5	-61.5	200	Trinidad	
2	9.5	-85.5	1500	San Jose	Known
3	-12.03333	-77.03333	1000	Lima	Stations
4	0	-89.00	100	Galapagos	Unknown
					Station
I	-11.8	-72.4	3200 000		
II	4.0	-64.4	3200 000		Satellite
III	12.0	-86.5	3200 000		Positions

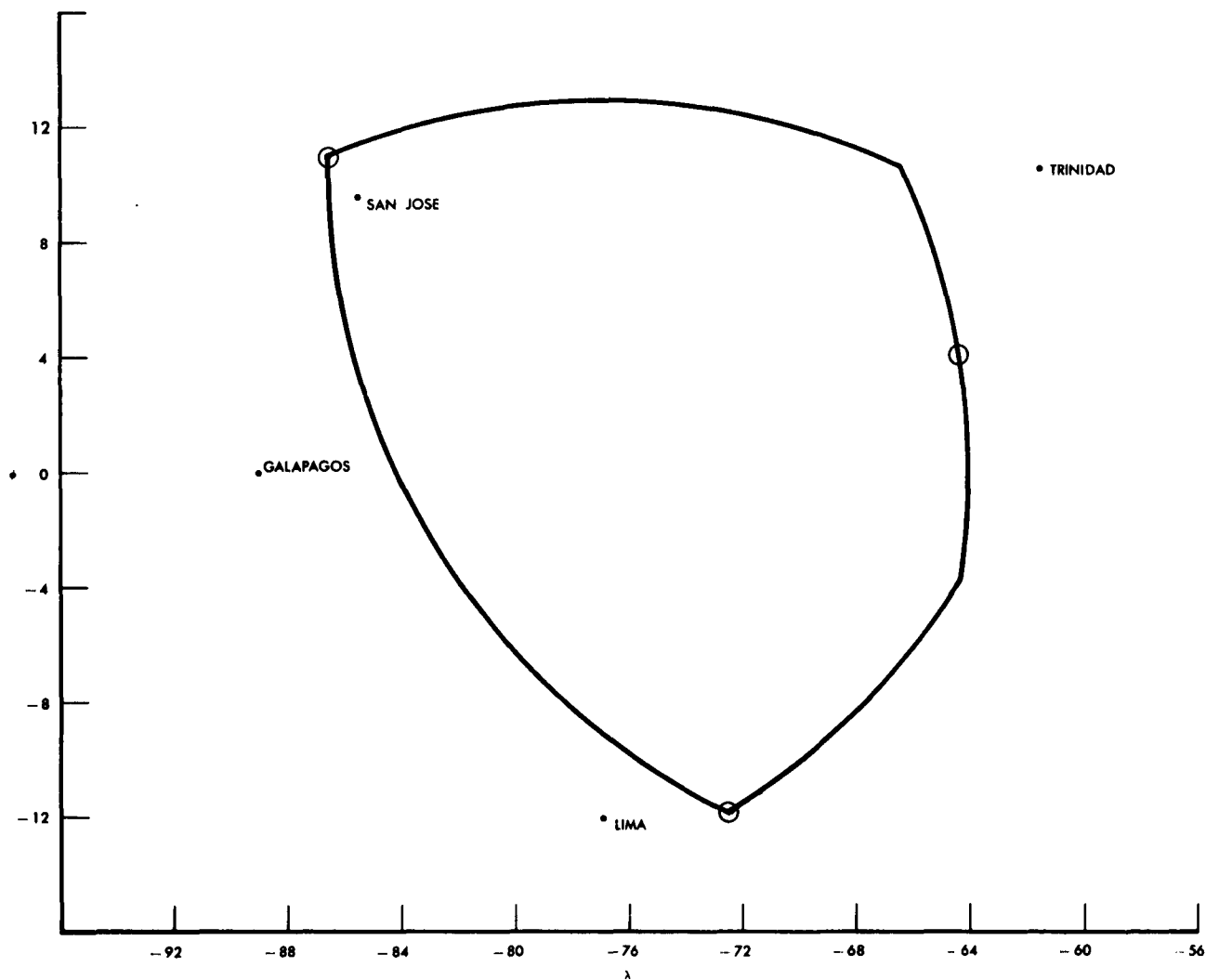
Limit 60° Zenith Distance

FINAL RESULTS:

Galapagos  $\pm$  123.3 Uncertainty (meters)

SATELLITE D, PHASE III-5  
LIMIT 60° ZENITH DISTANCE  
TRINIDAD, SAN JOSE, LIMA—  
(KNOWN)  
GALAPAGOS  
(UNKNOWN)

SATELLITE POSITIONS FIXED BY THE  
3 KNOWN STATIONS AND THE UNKNOWN  
STATION FOR THE SIMULTANEOUS OBSERVATION



90° Inclination  
 SATELLITE D Height = 3200 KM

Phase III - 6

Memphis, Haiti, Manganillo --- Galapagos, Clipperton, Bermuda

	$\phi$	$\lambda$	h(meters)	Name	
1	34.71667	-89.75	50	Memphis	
2	19.0	-72.0	500	Haiti	Known Stations
3	19.5	-104.0	200	Manganillo	
4	0	-89.0	100	Galapagos	Unknown Station
I	9.6	-88.0	3200 000		
II	24.0	-96.4	3200 000		Satellite Positions
III	23.0	-79.4	3200 000		
4	10.5	-106.33333	100	Clipperton	Unknown Station
I	15.2	-96.8	3200 000		
II	32.0	-93.4	3200 000		Satellite Positions
III	12.8	-81.4	3200 000		
4	32.31667	-64.83333	100	Bermuda	Unknown Station
I	37.1	-89.2	3200 000		
II	26.4	-80.0	3200 000		Satellite Positions
III	12.6	-80.0	3200 000		

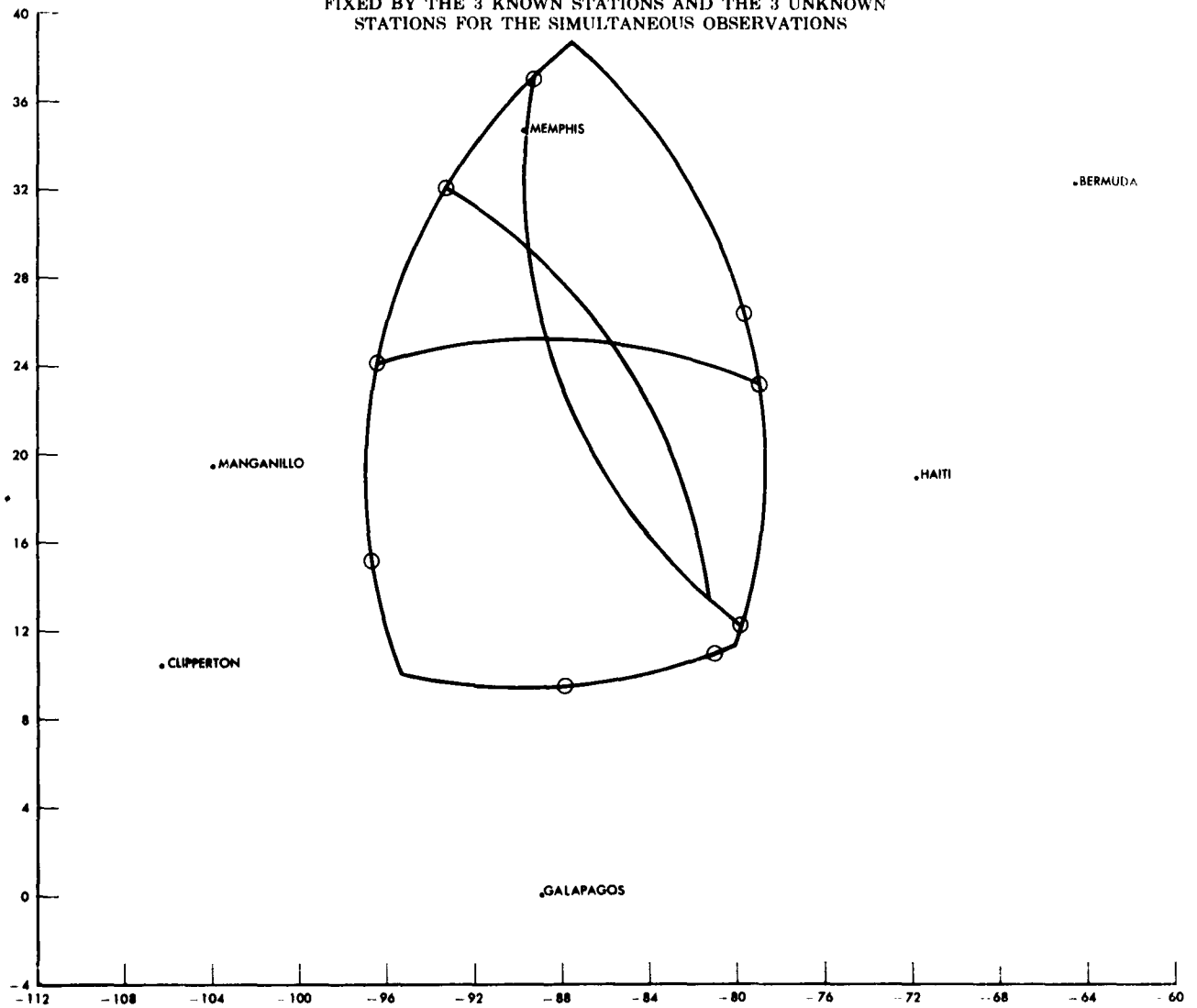
Limit - 60° Zenith Distance

FINAL RESULTS:

Galapagos	$\pm$ 189.8	Uncertainty (meters)
Clipperton	$\pm$ 174.8	" "
Bermuda	$\pm$ 430.8	" "

SATELLITE D, PHASE III-6  
 LIMIT 60° ZENITH DISTANCE  
 MEMPHIS, HAITI, MANGANILLO  
 (KNOWN)  
 GALAPAGOS, CLIPPERTON, BERMUDA  
 (UNKNOWN)

STATION COORDINATES AND SATELLITE POSITIONS  
 FIXED BY THE 3 KNOWN STATIONS AND THE 3 UNKNOWN  
 STATIONS FOR THE SIMULTANEOUS OBSERVATIONS





SATELLITE D      90° Inclination  
 Height = 3200 KM

Phase IV - 1

Hawaii, Midway, Howland --- Christmas, Canton

	$\phi$	$\lambda$	h(meters)	Name	
1	19.21667	-155.86667	457	Hawaii	
2	28.21667	-177.33333	3	Midway	Known Stations
3	0.8	-176.63333	9	Howland	
4	2.0	-157.5	2	Christmas	Unknown Station
I	3.5	-179.2	3200 000		
II	14.4	-152.6	3200 000		Satellite Positions
III	22.0	-176.4	3200 000		
4	-2.5	-171.8	10	Canton	Unknown Station
I	3.5	-179.2	3200 000		
II	14.4	-152.6	3200 000		Satellite Positions
III	22.0	-176.4	3200 000		

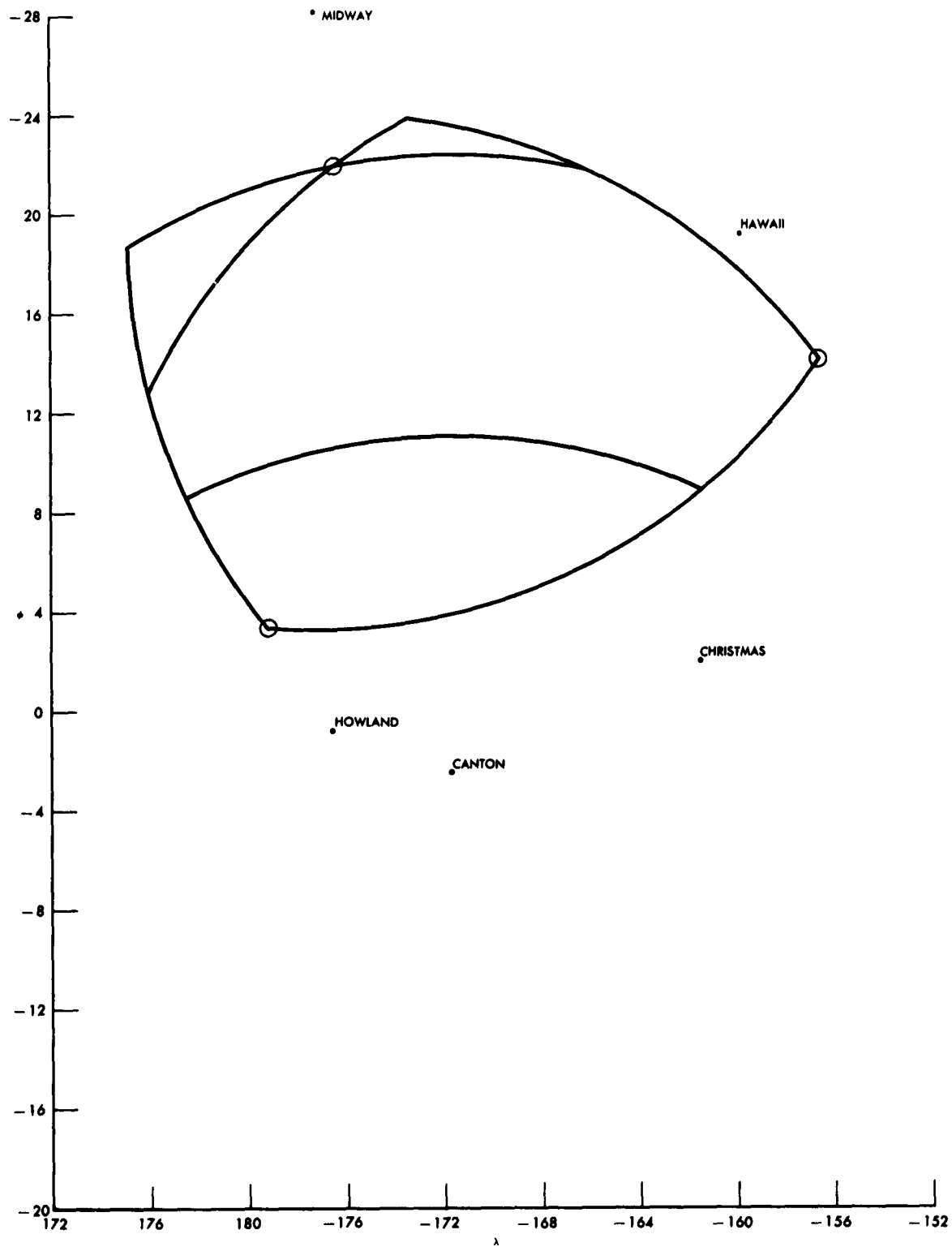
Limit 60° Zenith Distance

FINAL RESULTS:

Christmas  $\pm$  146.5 Uncertainty (meters)  
 Canton  $\pm$  133.6 " "

SATELLITE D, PHASE IV-1  
 LIMIT 60° ZENITH DISTANCE  
 HAWAII, MIDWAY, HOWLAND—  
 (KNOWN)  
 CHRISTMAS, CANTON  
 (UNKNOWN)

STATION COORDINATES AND SATELLITE POSITIONS  
 FIXED BY THE 3 KNOWN AND 2 UNKNOWN STATIONS  
 FOR THE SIMULTANEOUS OBSERVATIONS



SATELLITE D 90° Inclination

Height = 3200 KM

Phase IV - 2

Howland, Christmas, Johnston --- Upolu, Tonga, Papeete

	$\phi$	$\lambda$	h(meters)	Name	
1	0.8	-176.63333	9	Howland	
2	1.98333	-157.48333	2	Christmas	Known Stations
3	16.75	-169.51667	12	Johnston	
4	-13.6667	-171.7	18	Upolu	Unknown Station
I	9.2	178.7	3200 000		
II	-8.1	-168.0	3200 000		Satellite Positions
III	1.6	-152.0	3200 000		
4	-21.26667	-175.11667	49	Tonga	Unknown Station
I	2.4	177.6	3200 000		
II	-8.0	-171.2	3200 000		Satellite Positions
III	-4.9	-156.4	3200 000		
4	-17.55	-149.56667	0	Papeete	Unknown Station
I	-7.7	-172.6	3200 000		
II	-5.6	-157.6	3200 000		Satellite Positions
III	7.0	-152.8	3200 000		

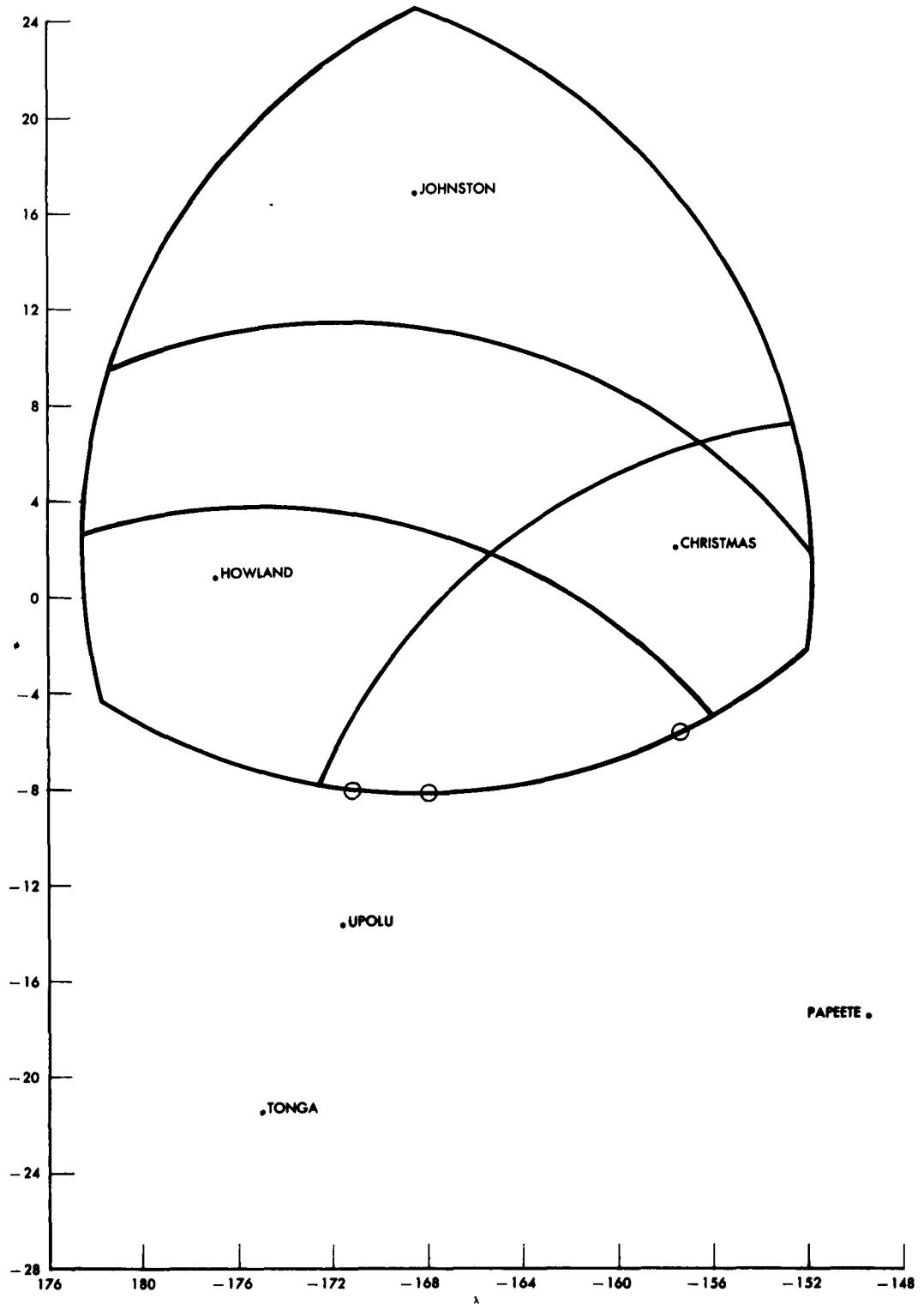
Limit 60° Zenith Distance

FINAL RESULTS:

Upolu  $\pm$  190.6 Uncertainty (meters)  
Tonga  $\pm$  414.4 " "  
Papeete  $\pm$  396.8 " "

SATELLITE D, PHASE IV-2  
 LIMIT 60° ZENITH DISTANCE  
 HOWLAND, CHRISTMAS, JOHNSTON  
 (KNOWN)  
 UPOLU, TONGA, PAPEETE  
 (UNKNOWN)

STATION COORDINATES AND SATELLITE POSITIONS  
 FIXED BY THE 3 KNOWN AND 3 UNKNOWN STATIONS  
 FOR THE SIMULTANEOUS OBSERVATIONS



90° Inclination  
SATELLITE D  
Height = 3200 KM

Phase IV - 3

Nauru, Howland, Tonga - - - Nandi, Guadalcanal, New Caledonia

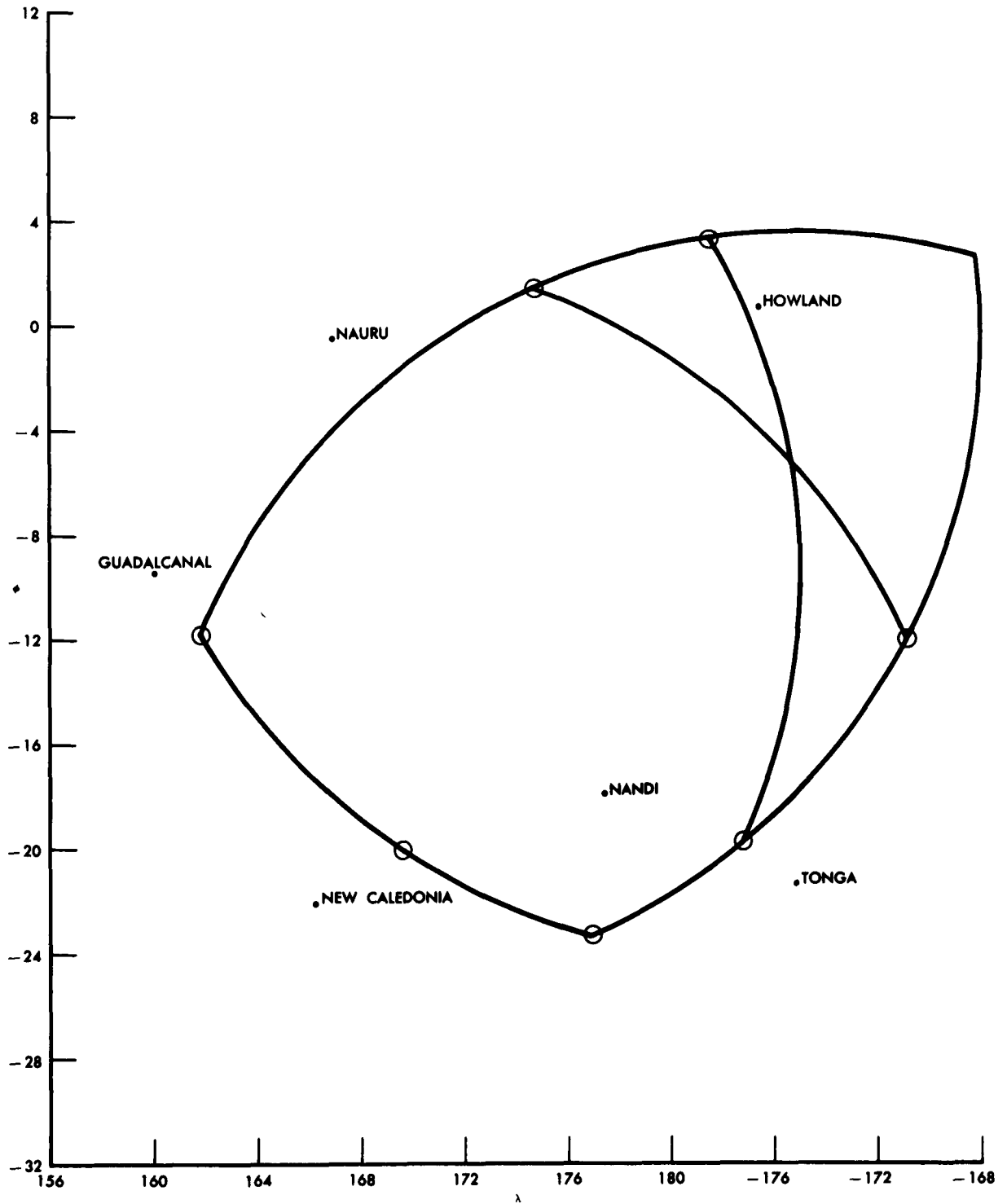
	$\phi$	$\lambda$	h(meters)	Name	
1	-0.53333	166.91667	8	Nauru	
2	0.8	-176.63333	9	Howland	Known Stations
3	-21.26667	-175.11667	49	Tonga	
4	-17.9	177.45	19	Nandi	Unknown Station
I	-11.8	161.9	3200 000		
II	-23.2	177.0	3200 000		Satellite Positions
III	2.6	-168.4	3200 000		
4	-9.43333	159.96667	6	Guadalcanal	Unknown Station
I	-11.8	161.9	3200 000		
II	-20.0	-176.5	3200 000		Satellite Positions
III	2.8	-178.7	3200 000		
4	-22.05	166.18333	17	New Caledonia	Unknown Station
I	-20.0	169.6	3200 000		
II	-12.0	-171.0	3200 000		Satellite Positions
III	1.3	174.8	3200 000		

Limit 60° Zenith Distance

FINAL RESULTS:

Nandi	± 104.0	Uncertainty (meters)
Guadalcanal	± 169.2	" "
New Caledonia	± 190.9	" "

SATELLITE D, PHASE IV-3  
 LIMIT 60° ZENITH DISTANCE  
 NAURU, HOWLAND, TONGA  
 (KNOWN)  
 NANDI, GUADALCANAL, NEW CALEDONIA  
 (UNKNOWN)  
 STATION COORDINATES AND SATELLITE POSITIONS  
 FIXED BY THE 3 KNOWN AND 3 UNKNOWN STATIONS  
 FOR THE SIMULTANEOUS OBSERVATIONS



SATELLITE D      90° Inclination  
Height = 3200 KM

Phase IV - 4

Nandi, Guadalcanal, Brisbane - - Auckland, Wellington

	$\phi$	$\lambda$	h(meters)	<u>Name</u>	
1	-17.9	177.45	19	Nandi	
2	-9.43333	159.96667	6	Guadalcanal	Known Stations
3	-27.5	152.83333	150	Brisbane	
4	-36.86667	174.8	30	Auckland	Unknown Station
I	-32.4	169.6	3200 000		
II	-24.2	153.5	3200 000		Satellite Positions
III	-12.2	172.4	3200 000		
4	-39.16667	174.78333	31	Wellington	Unknown Station
I	-32.4	169.6	3200 000		
II	-25.6	153.9	3200 000		Satellite Positions
III	-14.4	174.0	3200 000		

Limit - 60° Zenith Distance

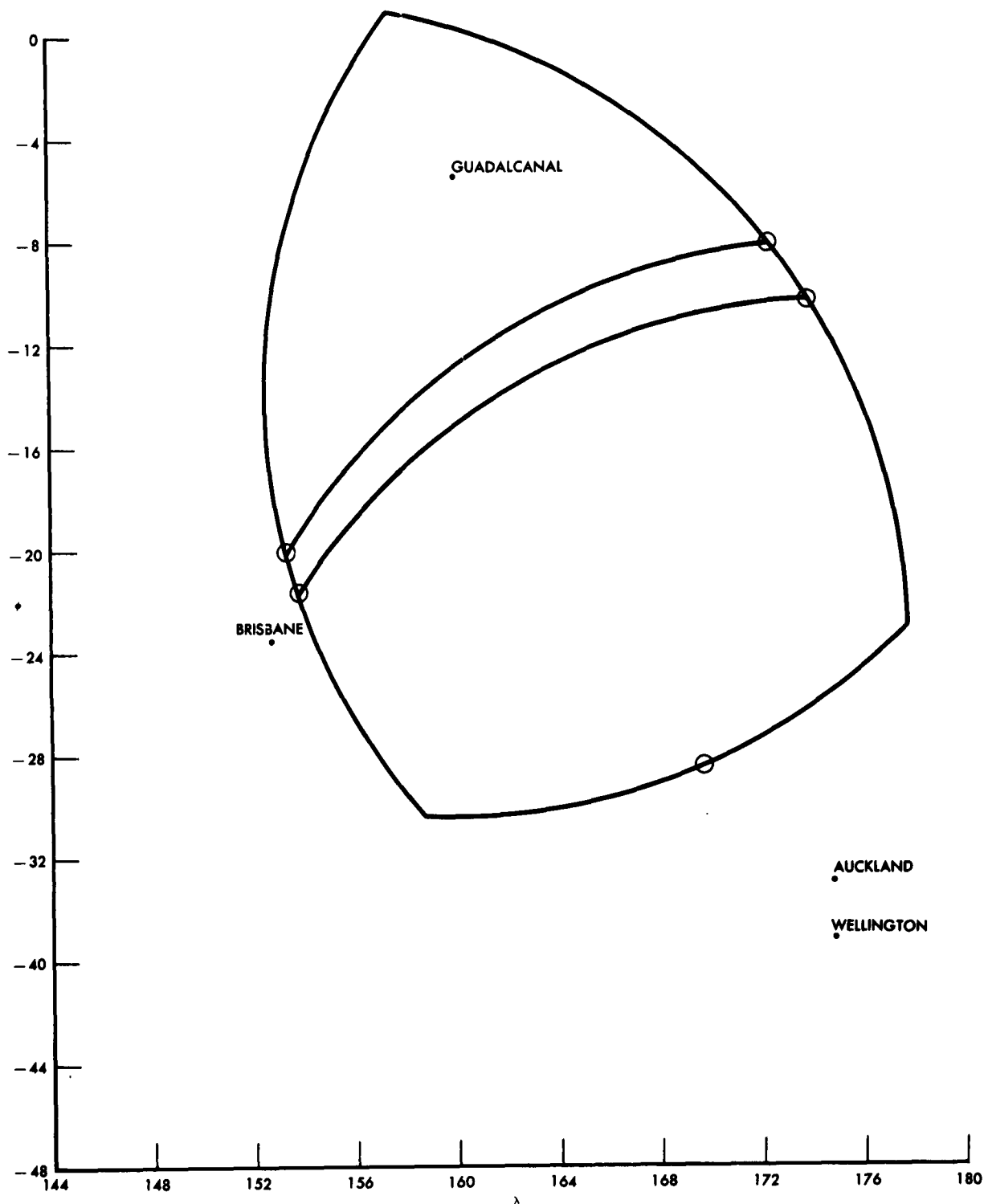
FINAL RESULTS:

Auckland  $\pm$  210.6    Uncertainty (meters)

Wellington  $\pm$  243.5    "        "

SATELLITE D, PHASE IV-4  
 LIMIT 60° ZENITH DISTANCE  
 NANDI, GUADALCANAL, BRISBANE  
 (KNOWN)  
 AUCKLAND, WELLINGTON  
 (UNKNOWN)

STATION COORDINATES AND SATELLITE POSITIONS  
 FIXED BY THE 3 KNOWN AND 2 UNKNOWN STATIONS  
 FOR THE SIMULTANEOUS OBSERVATIONS





SATELLITE D 90° Inclination  
Height = 3200 KM

Phase V - 1

Sidi Ifni, Tunis, Brest - - - Pico

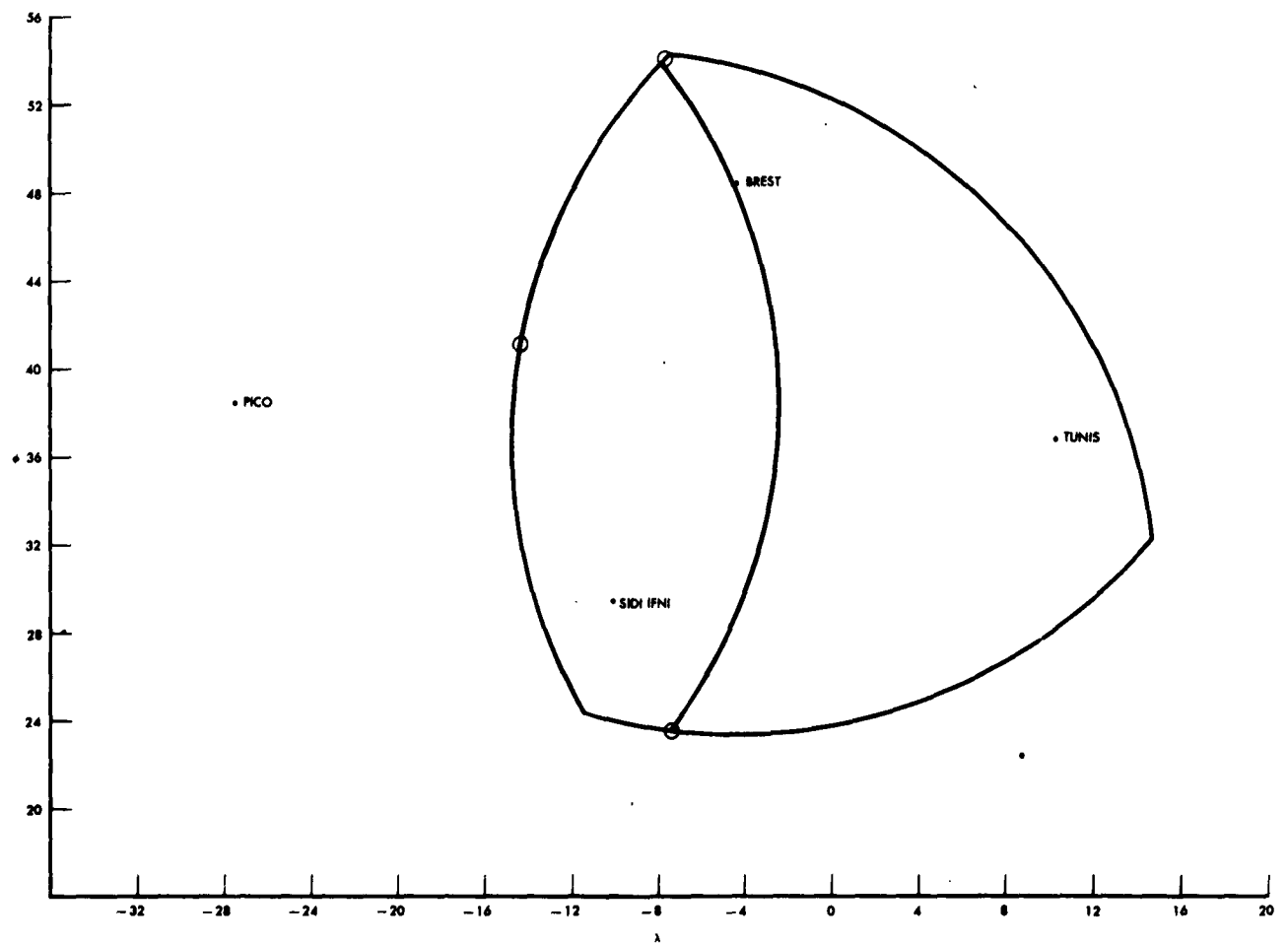
	$\phi$	$\lambda$	h(meters)	<u>Name</u>	
1	29.38333	-10.18333	49	Sidi Ifni	
2	36.8	10.18333	15	Tunis	Known Stations
3	48.4	-4.48333	91	Brest	
4	38.05	-27.05	305	Pico	Unknown Station
I	41.2	-14.4	3200 000		
II	53.0	-8.0	3200 000		Satellite Positions
III	23.6	-7.6	3200 000		

Limit 60° Zenith Distance

FINAL RESULTS:

Pico  $\pm$  360.7 Uncertainty (meters)

SATELLITE D, PHASE V-1  
 LIMIT 60° ZENITH DISTANCE  
 SIDI IFNI, TUNIS, BREST-PICO  
 (KNOWN) (UNKNOWN)  
 STATION COORDINATES AND SATELLITE POSITIONS  
 FIXED BY THE 3 KNOWN AND 1 UNKNOWN  
 STATION FOR THE SIMULTANEOUS OBSERVATION



SATELLITE D      90° Inclination  
 Height = 3200 KM

Phase V - 2

Molde, Kenmare, Reykjavik - - - Jan Mayen

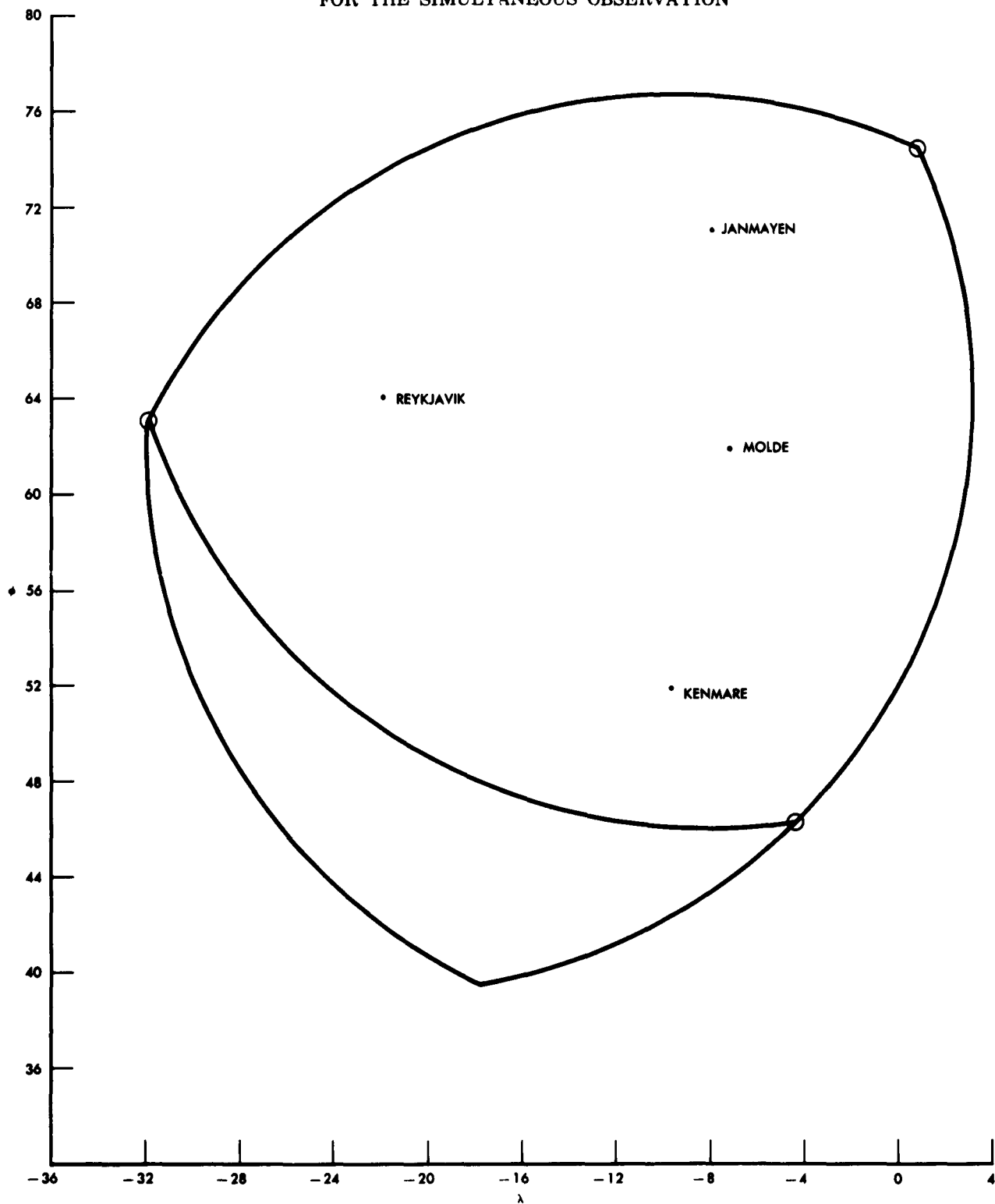
	$\phi$	$\lambda$	h(meters)	<u>Name</u>	
1	62.06667	-7.21667	914	Molde	
2	51.88333	-9.58333	305	Kenmare	Known Stations
3	64.13333	-21.91667	0	Reykjavik	
4	71.13333	-8.03333	305	Jan Mayen	Unknown Station
I	46.4	-4.3	3200 000		
II	63.2	-31.8	3200 000		Satellite Positions
III	74.6	0.8	3200 000		

FINAL RESULTS:

Jan Mayen  $\pm$  167.4 Uncertainty (meters)

SATELLITE D, PHASE V-2  
 LIMIT 60° ZENITH DISTANCE  
 MOLDE, KENMARE, REYKJAVIK—JAN MAYEN  
 (KNOWN) (UNKNOWN)

STATION COORDINATES AND SATELLITE POSITIONS  
 FIXED BY THE 3 KNOWN AND 1 UNKNOWN STATIONS  
 FOR THE SIMULTANEOUS OBSERVATION



SATELLITE D      90° Inclination  
 Height = 3200 KM

Phase V - 3

Narvik, Helsinki, Shetland - - - Sorkaap

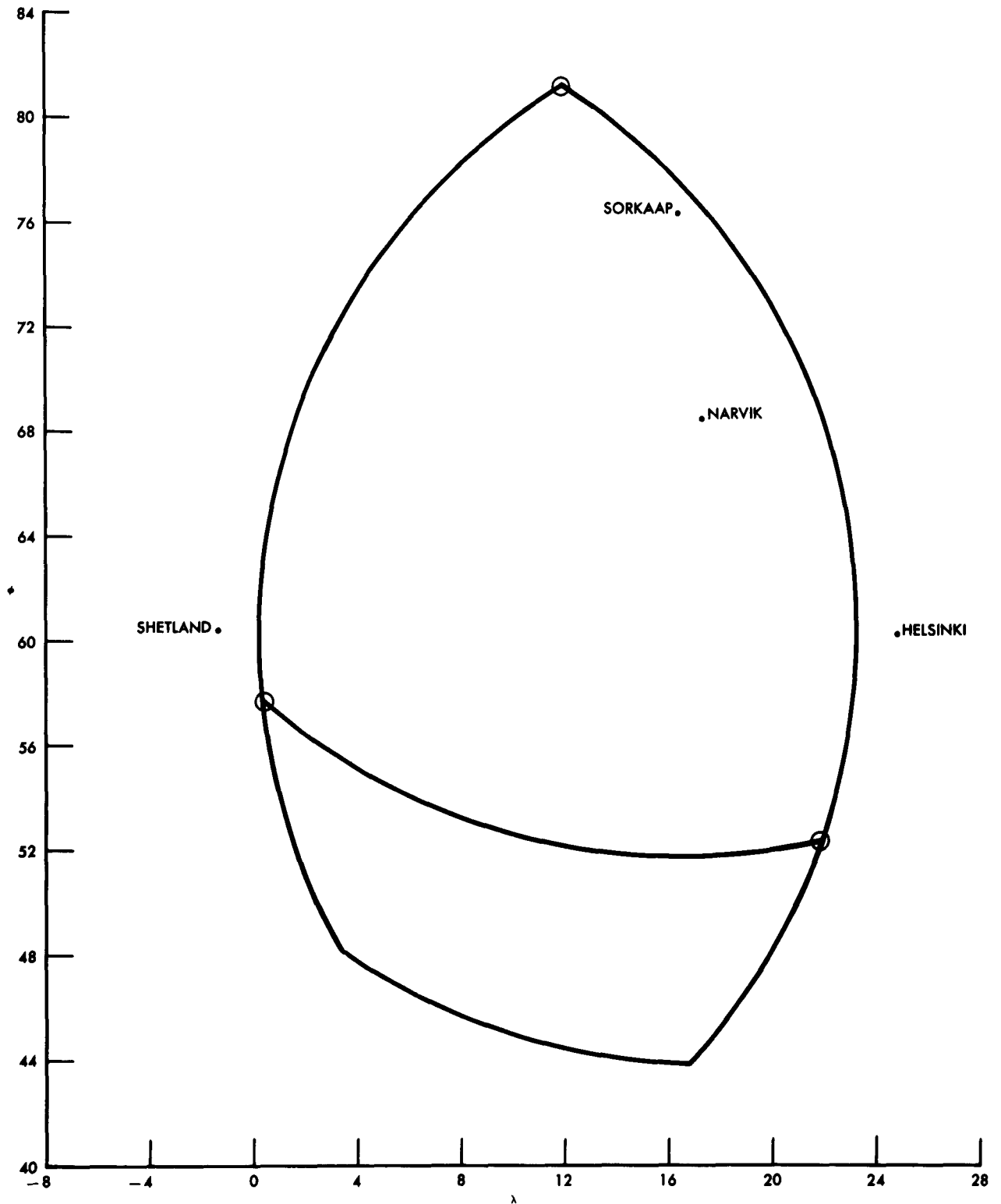
	$\phi$	$\lambda$	h(meters)	Name	
1	68.41667	17.41667	1448	Narvik	
2	60.16667	24.93333	123	Helsinki	Known Stations
3	60.38333	-1.35	152	Shetland	
4	76.25	16.5	200	Sorkaap	Unknown Station
I	57.8	0.6	3200 000		
II	54.5	21.9	3200 000		Satellite Positions
III	80.9	12.0	3200 000		

Limit 60° Zenith Distance

FINAL RESULTS:

Sorkaap       $\pm$  223.6 Uncertainty (meters)

SATELLITE D, PHASE V-3  
 LIMIT 60° ZENITH DISTANCE  
 NARVIK, HELSINKI, SHETLAND-SORKAAP  
 (KNOWN) (UNKNOWN)  
 STATION COORDINATES AND SATELLITE POSITIONS  
 FIXED BY THE 3 KNOWN AND 1 UNKNOWN STATIONS  
 FOR THE SIMULTANEOUS OBSERVATION



SUMMARY  
FINAL RESULTS

Satellite	Phase No.	Known Sites	Unknown Sites	Uncertainty (meters)
A	I - 1	Johnston Kauai Laysan	Tarawa	± 99.3
A	I - 2	Johnston Kauai Hawaii	Palmyra Howland	584.8 157.8
A	I - 3	Wake Eniwetok Maloelap	Midway Howland	102.2 195.4
A	I - 4	Wake Eniwetok Maloelap	Tarawa Nauru Kusaie	407.1 521.8 160.0
A	I - 5	Wake Eniwetok Maloelap	Tokyo Ponape Marcus	103.4 159.0 141.8
A	I - 6	Kusaie Ponape Eniwetok	Truk	229.2
C	I - 7	Truk Ponape Kapingamarangi	Pulusuk Ifalik	126.2 262.1
C	I - 8	Truk Ifalik Manus	Saipan Guam Ulithi	281.9 215.4 193.9
C	I - 9	Truk Ifalik Manus	Palau New Guinea	369.9 297.2
C	I - 10	Kapingamarangi Rabaul Manus	Port Moresby Cooktown Townsville	209.1 292.9 288.5
C	I - 11	Ulithi Babelthmap Kaap D'Urville	Catabato Surigao	589.6 509.5

SUMMARY  
FINAL RESULTS

Satellite Phase No. Known Sites Unknown Sites Uncertainty  
(meters)

B	I - 12	Kaap D'Urville Kuching Aparri	Darwin Wyndham	+ 181.6 - 203.1
C	II - 1	Laccadives Maldives Columbo	Cocos Island	393.5
B	II - 2	Karachi Columbo Calcutta	Chagos Archipelago Dante Soetra	215.0 373.2 235.1
B	II - 3	Dante Chagos Arch. Mangalore	Mogadiscio Diego Suarez Mauritius	155.7 248.9 309.7
B	II - 4	Chagos Arch. Mogadiscio Mauritius	Mombasa Fort Dauphin Kilwa	156.8 173.1 157.6
B	II - 5,6	Lusaka Brazzaville Luderitz	Brazzaville Saint Helena	267.8 231.0
B	II - 7	Brazzaville Saint Helena Walvisbaa	Ascension Accra Lagos	179.4 204.9 180.4
B	II - 8	Ascension Brazzaville Accra	Freetown Dakar Boa Vista	148.7 158.5 202.1
B	II - 9	Accra Brazzaville Ascension	Tetuan Oran	234.6 199.4
B	III - 1	Ascension Accra Dakar	Fernando de Noronha Recife	200.5 312.5
C	III - 2	Fernando de Noronha Parnaiba Aracaju	Freetown	151.1



SUMMARY  
FINAL RESULTS

Satellite	Phase No.	Known Sites	Unknown Sites	Uncertainty (meters)
D	III - 3	Arica Buenos Aires Rio de Janeiro	Palmer Peninsula Punta Arenas Falkland Islands	+ 435.5 - 410.7 261.9
D	III - 4	Parnaiba Rio de Janeiro Arica	Trinidad Buenos Aires	265.6 194.6
D	III - 5	Trinidad San Jose Lima	Galapagos	123.3
D	III - 6	Memphis Haiti Manganillo	Galapagos Clipperton Bermuda	189.8 174.8 430.8
D	IV - 1	Hawaii Midway Howland	Christmas Canton	146.5 133.6
D	IV - 2	Howland Christmas Johnston	Upolu Tonga Papeete	190.6 414.4 396.8
D	IV - 3	Nauru Howland Tonga	Nandi Guadalcanal New Caledonia	104.0 169.2 190.9
D	IV - 4	Nandi Guadalcanal Brisbane	Auckland Wellington	210.6 243.5
D	V - 1	Sidi Ifni Tunis Brest	Pico	360.7
D	V - 2	Molde Kenmare Reykjavik	Jan Mayen	167.4
D	V - 3	Narvik Helsinki Shetland	Sorkaap	223.6

#### V. SUMMARY OF RESULTS

The probable error in the corrections to the unknown positions is a function of the relationships between the three known stations and the area of simultaneity; relatively large errors indicate that the stations were poorly positioned. In this case a longer observational period would be required in order to obtain a smaller probable error.

#### VI. CONCLUSION

The determination of the coordinates of an unknown geodetic point on the earth in relation to other known points can be accomplished through the use of satellites with known heights and inclinations. Given the geometry set by the three points whose positions are known and the unknown point, the error in the corrections to the position determined for the unknown point can be computed by applying the principle of propagation of errors. The results of this error analysis are affected to a great extent by the geometric configurations chosen and by the type of satellite-tracking equipment employed. For a more accurate determination of the geodetic positions, the equipment and geometric configurations used must be improved.

This report is intended to present a sample world-wide satellite tracking program and to give the results which could be expected from such a program if it were to be carried out. The program presented is not to be considered as a final plan.

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